

SALTUS GRAMMAR SCHOOL

AP Physics II



"The defibrillator's not working! Quick, everyone scuff your feet on the carpet!"

Unit 13 – Electrostatics

Name:

Date:

Summary

This unit deals with the effects of electrical charges. An electric charge has an electrical field surrounding it that affects other electrical charges around it. This has many applications in everyday life – from spray, sparks, lightning, photocopying and ink jet printers. The force that acts between charged particles is fundamental to chemistry, atomic and nuclear physics.

- 13.1 Charging
- 13.2 Coulomb's Law
- 13.3 Electric Field Strength
- 13.4 Electric Potential
- 13.5 Uniform Electrical Fields
- 13.6 Capacitance

There are two detailed labs to be performed during this topic and several pages of homework assignments. Additionally, I have included some past AP problems at the end of the topic.

While I have included spaces for you to make notes, answer questions and solve problems, it is expected that you make additional notes, comments and aide memoirs in any available space and annotate diagrams and equations as you desire. This will assist in future study endeavours as annotating texts etc is a valuable study skill.

Key Equations

13.1 Charges and Charging

Objectives:

- To know that electric charge is carried by the electron and proton and that they have a specific and discrete charge, e .
- To understand that macroscopic charge is due to electrons being moved from one place to another.
- To know how an object can be charged

Notes:

This section deals with stationary electric charge in contrast with our previous section on Gravitational fields that dealt with stationary and moving masses. Mass is not a discrete quantity. While it is true that protons, neutrons and other fundamental particles have discrete rest masses that affects of binding energy and relativity stop us from discussing mass as a discrete quantity. Charge, however, is discrete. All charges are multiples of 1.6×10^{-19} C, this is the electronic charge. Protons have a charge of exactly $+1e$ and electrons have a charge of exactly $-1e$. Charge is measured in Coulombs, symbol C.

All atoms in their normal state are electrically neutral. When atoms combine their outer electrons are used for chemical bonding, they are subsequently held in place. In some metals a lone outer electron may not be used in bonding and will hence be 'free'. These free electrons are capable of moving randomly from atom to atom as long as the atoms are still electrically neutral. The movement of these 'charge carriers' is electrical current. Materials with charge carriers are electrical conductors and materials without charge carriers are electrical insulators.

Charging by Contact (friction)

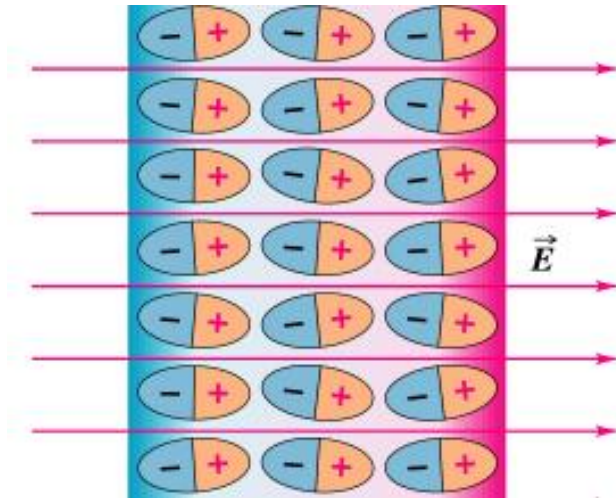


When wool and polythene are rubbed together they both become charged. Some of the outer electrons on the wool are physically scrapped off and deposited onto the polythene. Repeating the experiment with other materials allows us to ascertain that there are two types of charge, positive and negative. This is again another difference to mass where we only ever find positive values. It should be noted that anti matter has a positive mass but the opposite electrical charge to normal matter.

At no time during these simple experiments is charge created, it is simply transferred from one object to another. The conservation of charge in any reaction is absolute and fundamental to all physics. Energy and mass are conserved but only as a joint form.

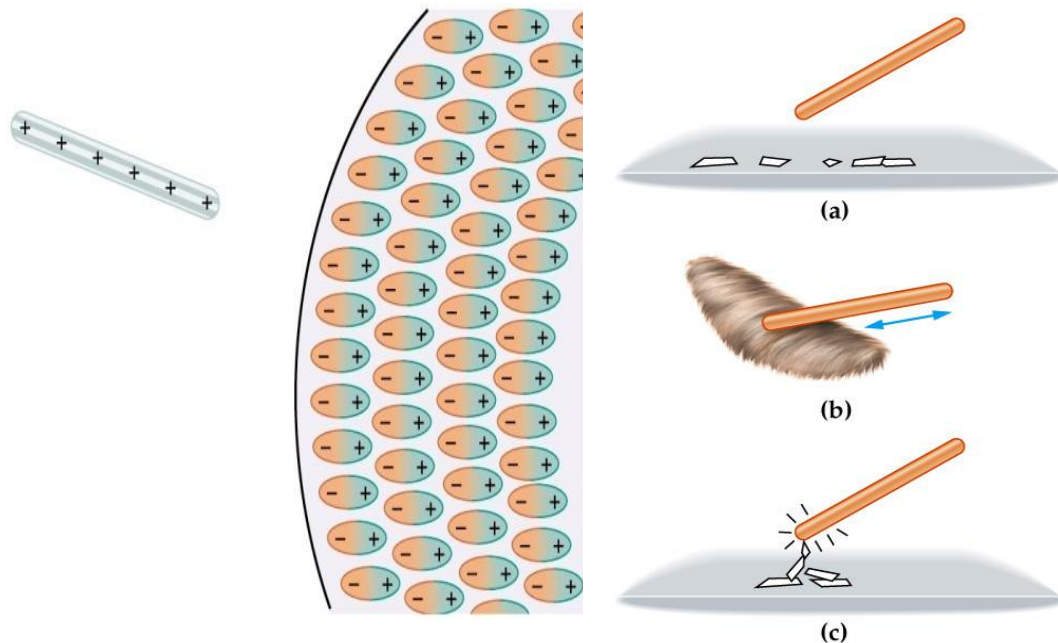
Polarisation and attracting neutral objects

When charging by induction the objects never physically touch. Initially a charged object (a negative rod in this case) is brought near a neutral object. Our neutral object is composed of a metallic conducting sphere placed upon an insulated stand.

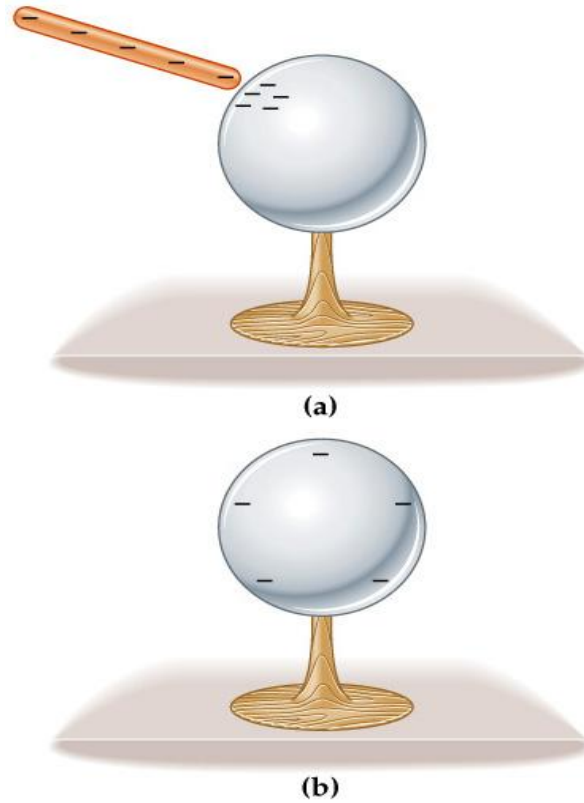


Electrons are repelled to the opposite side of the conducting sphere creating an effect called surface polarisation. Other than the closest and furthest layers of atoms which have a surplus of charge, all others cancel each other out and are neutral.

The net charge on this neutral object is still zero. The polarisation causes an imbalance of charge between different surfaces of the conductor.



When a charged object, such as a plastic rod, is brought near a neutral object, such as a small piece of paper, the paper is polarised. The neutral object is attracted to the charged object because the net charge on the surface closest to the charged object is opposite of the charged object. In this manner neutral conductors can be attracted to charged objects.

Charging by conduction

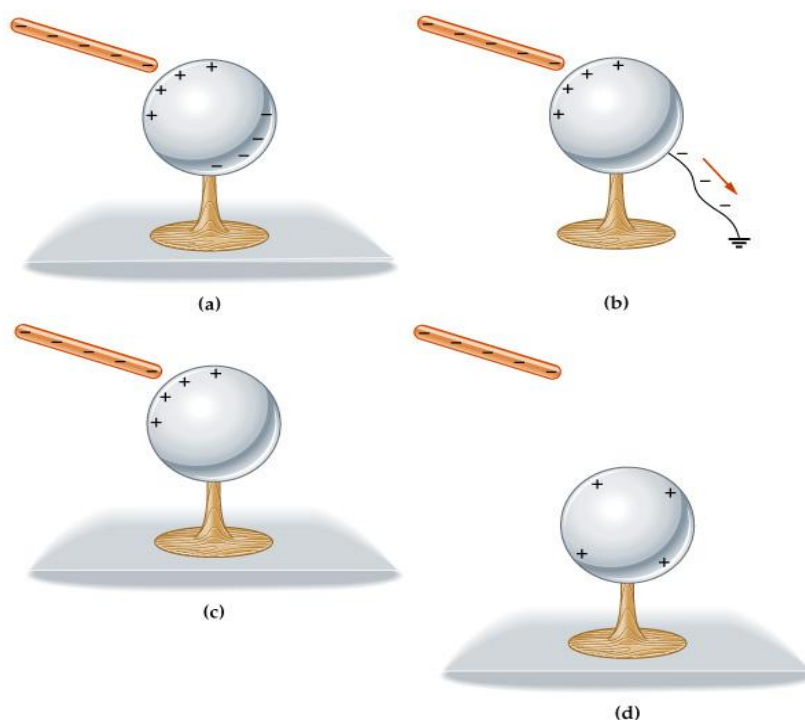
Charging by conduction occurs when a charged object comes into physical contact with a neutral object. Before contact the insulated metallic sphere has no charge while the rod has a net negative charge. When in contact the rod and sphere act as a single larger object. There is a net transfer of negative charge from the rod to the sphere until the forces of electric repulsion between excess charges are equal.

The rod is then removed from contact with the sphere. The sphere retains its net negative charge. The charge is equally dispersed over its entire surface so that the surface has the same electric potential.

When charged by conduction an object ends up with the the same charge as the original object. However the object that charged the sphere has lost some of its charge, so its own electric field will have decreased.

Charging by induction

1. Initially a charged rod is brought near a neutral conducting sphere. The charge of the rod will cause the sphere to become polarised, as shown in (a).
2. The second stage is to Earth the conducting sphere at which time the excess electrons will move to Earth, as shown in (b). The Earth connection can now be removed.
3. This leaves the conducting sphere with a net charge as shown in (c).
4. When the charged rod is moved away from the sphere the electrons move around the surface until the electric forces between them are equal, as shown in (d). The sphere has a net positive charge due to the loss of electrons.



Charging by conduction	Charging by induction
Charged object touches the conductor	Charged object does not touch the conductor.
Conductor ends up similarly charged to the object used to charge it	Conductor ends up oppositely charged to the object used to charge it.
The first charge is strong but gets weaker each time the conductor is recharged. (This is due to the original object giving up some charge every time it is touched.)	The first charge is strong and stays strong each time the conductor is recharged. (This is due to the original object not losing any charge in the process.)

13.2 Coulombs Law

Objectives:

- To know (learn!) the equation that describes the force that acts between two charges
- To be able to solve problems involving two or more charges interacting with each other.

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The magnitude of the force, F , between two electrically charged bodies, which are small compared with their separation, r , is inversely proportional to the product of their charges, Q_1 and Q_2 .

$$F = \frac{kQ_1Q_2}{r^2}$$

where for free space (or air) $k = 1/4\pi\epsilon_0$

Where:

k = constant.

ϵ_0 = Permittivity of free space (vacuum).

Permittivity

Unlike gravitational fields that can move through matter unaffected and cannot be shielded, electric forces are affected by the material through which they are propagating. The permittivity of a material can be conceptualised as a description of how hard or easy it is for an electrical field to move through the material.

The permittivity of free space can be calculated from one of Maxwell's equations, where:

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

Where:

c = Speed of light in a vacuum, $3 \times 10^8 \text{ m s}^{-1}$.

ϵ_0 = Permittivity of free space (vacuum).

μ_0 = Permeability of free space, $4\pi \times 10^{-7} \text{ H m}^{-1}$.

Example 1

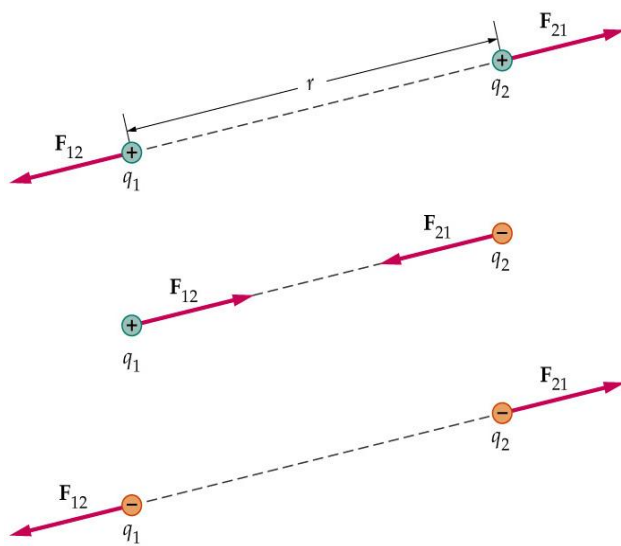
Given the data above calculate a value for the permittivity of free space, and hence a value for k .

Example 2

Calculate the electrical and gravitational forces between an electron and proton in a hydrogen atom if their separation is 5.3×10^{-11} m. Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg; mass of an electron, $m_e = 9.11 \times 10^{-31}$ kg.

Example 3

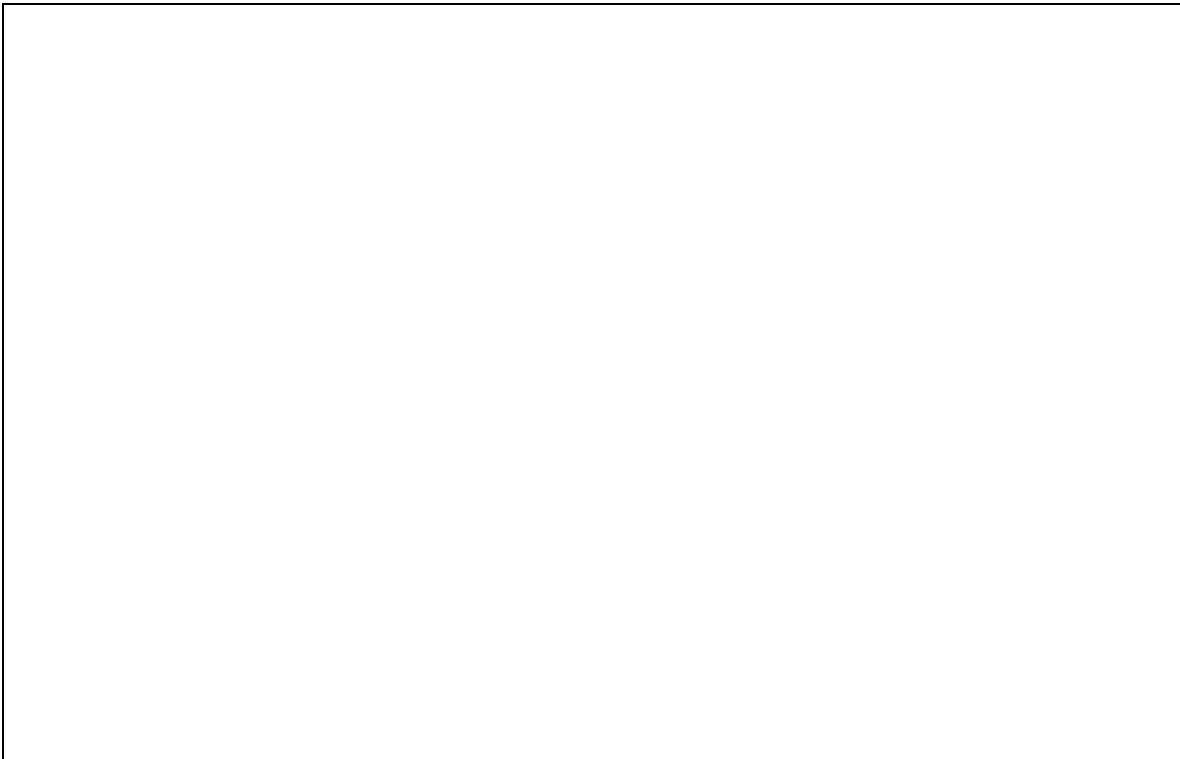
The Earth and the Sun have a slight electronic charge. Calculate the electrical and gravitational forces between them. Charge on Earth, $Q_E = 2$ MC; charge on Sun, $Q_S = 6000$ MC. Mass of Earth, $M_E = 6 \times 10^{24}$ kg; mass of Sun, $M_S = 2 \times 10^{30}$ kg. The distance between Earth and Sun is 1.5×10^{11} m.

Interaction of multiple charges

The vector nature of the electric force means that we have both positive and negative values.

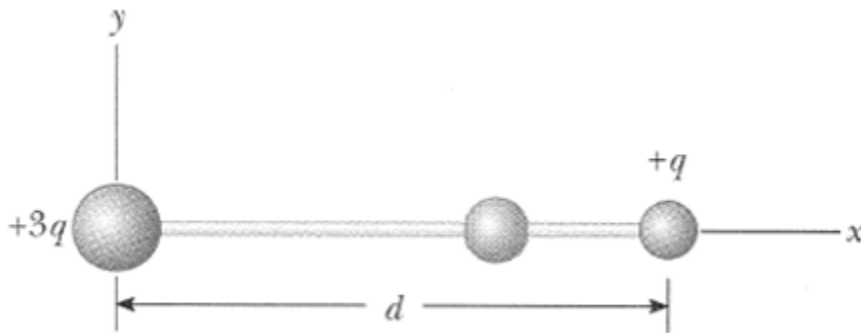
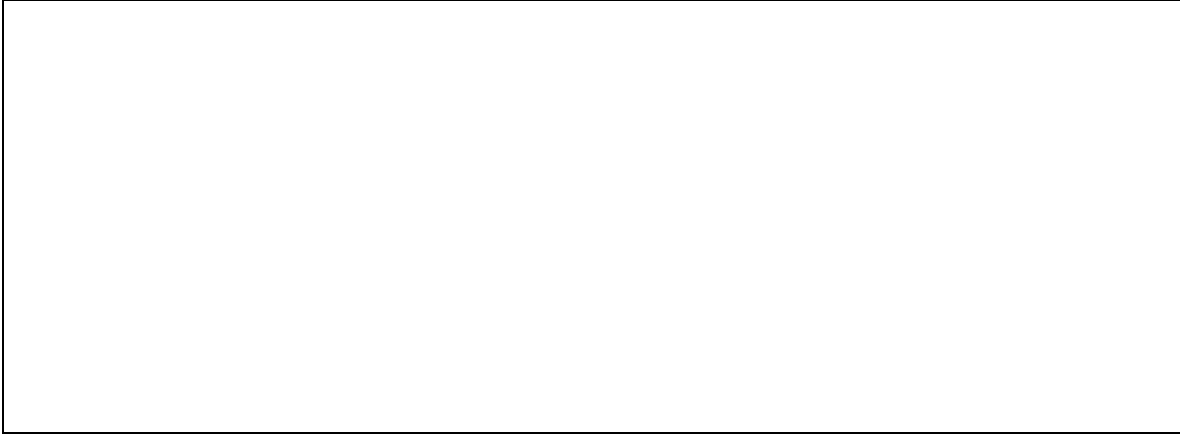
Example 4

Two charges, A and B, are fixed to a surface at a distance of 0.5 m. The charge on A is 15 nC and the charge on B is -10 nC. (a) Calculate the force on A from B. (b) Calculate the force from A and B on a point charge of magnitude 5 nC which is positioned midway between A and B.

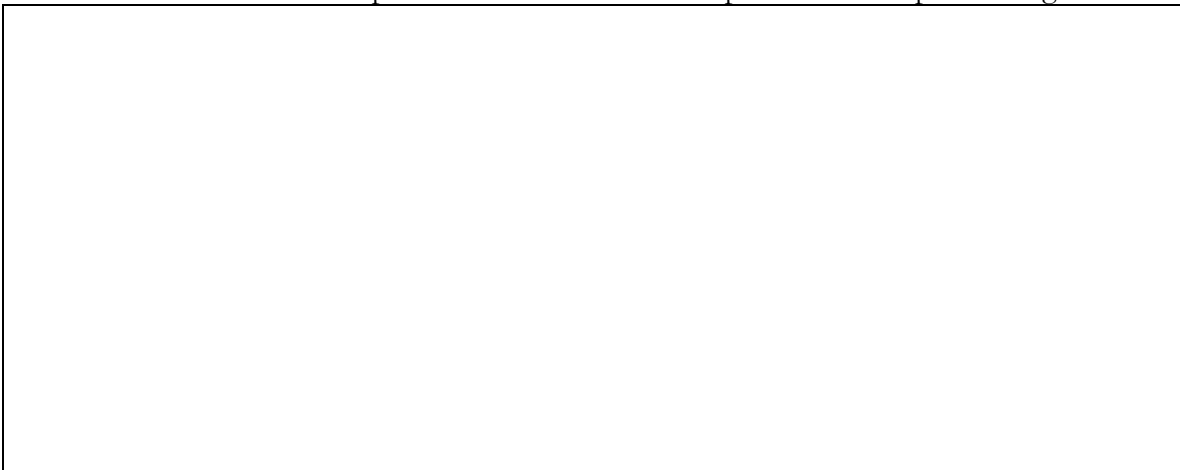


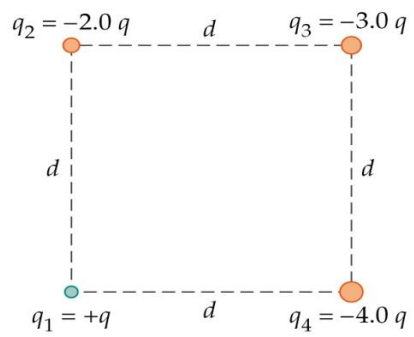
Example 5

Two charges, A and B, are fixed to a surface and separated by a distance d . The charge on A is Q and the charge on B is $3Q$. A point charge of magnitude q is placed between A and B so that the net force upon it is zero. Calculate the position of q .

**Example 6**

Two charges, A and B, are fixed to a surface at a distance of 0.4 m. The charge on A is $30 \mu\text{C}$ and the charge on B is $20 \mu\text{C}$. A point charge of magnitude $10 \mu\text{C}$ is placed between A and B so that the net force upon it is zero. Calculate the position of the point charge.



Example 7

Four charges are placed at the corners of a square are shown here. Calculate the force acting on q_1 .

13.3 Electric Field Strength

Objectives:

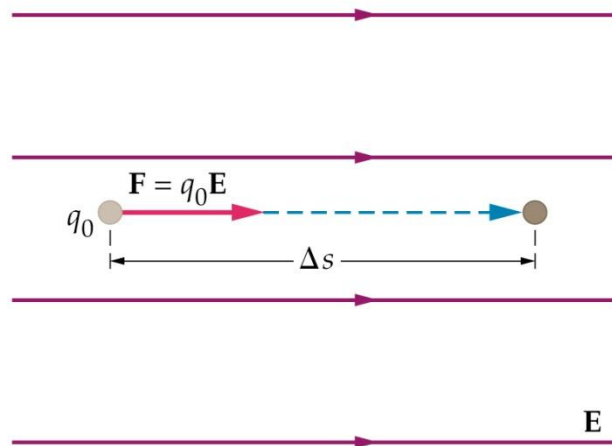
- To be able to describe the concept of electric fields.
- To be able to calculate the electric field strength surrounding a point charge.
- To be able to calculate the electric field strength between two charged plates.
- To be able to calculate the force on a point charge due to an electric field.

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An electric field is a region in which there are forces on charges. An electric field exists in a region if electrical forces are exerted on charged bodies in that region. The direction of an electric field at a point is the direction in which a small positive charge would move i.e. it moves from positive to negative. The electric field strength, E , at a point is defined as the force exerted by the field on a unit test charge placed at that point.

Electric Field Strength = Force \div Charge

$$E = \frac{F}{Q}$$



Question: Describe what an electric field is in your own words.

The electric field strength due to a point charge at a distance r can be given by:

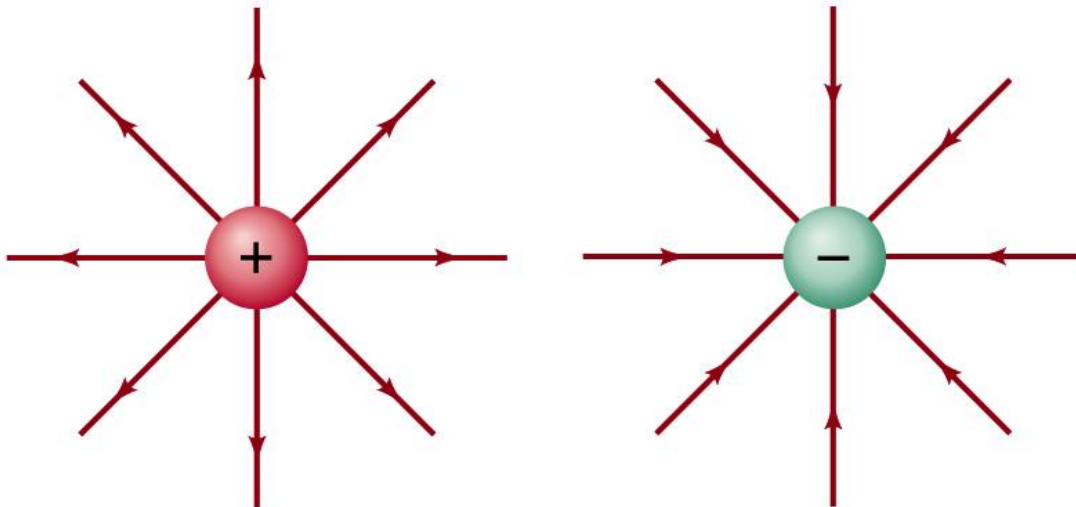
$$E = \frac{kQ}{r^2}$$

Electric field strength is a vector quantity – it has a direction. Electric field strength is the analogous quantity to gravitational field strength - the acceleration due to gravity. Once again field strength is inversely proportional to the square of the distance from the charge.

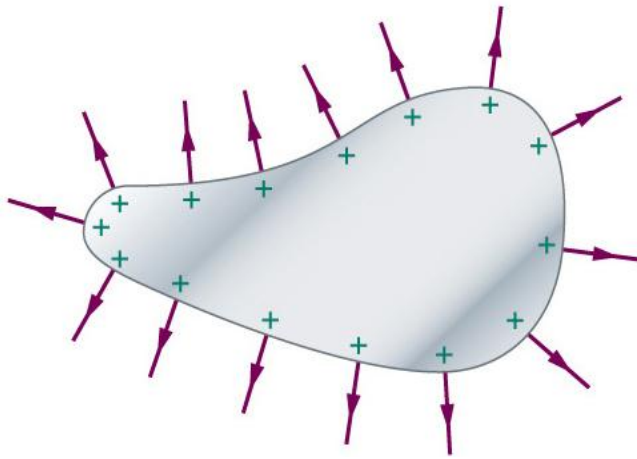
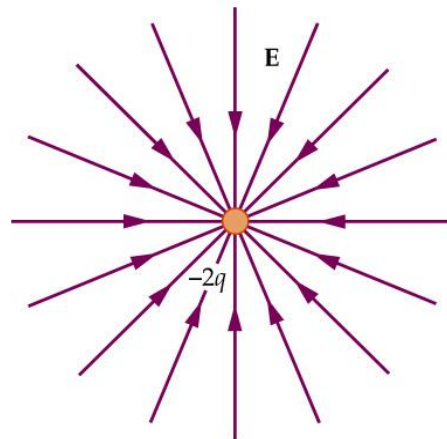
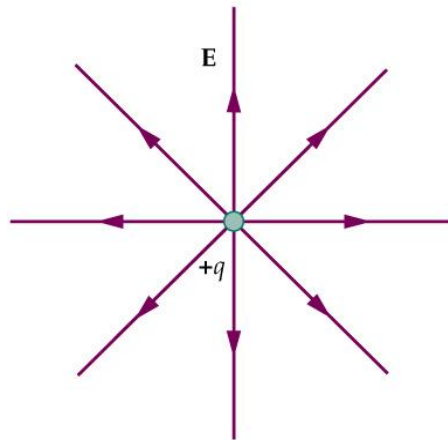
Question: Where does this inverse-square nature of the electric field come from? (Consider geometry)

Electric field around point charges

Electric field is a vector quantity. The positive direction is defined as the path taken by a positive point charge.



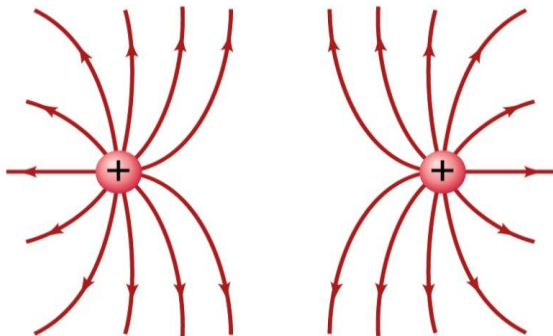
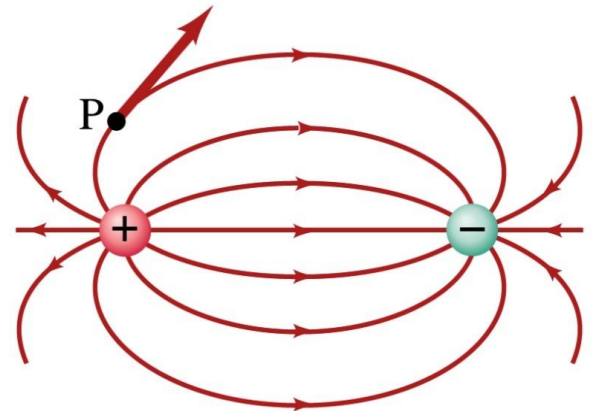
Electric field lines act in a radial direction from point charges and never cross each other. The stronger the field and closer the field lines will become.



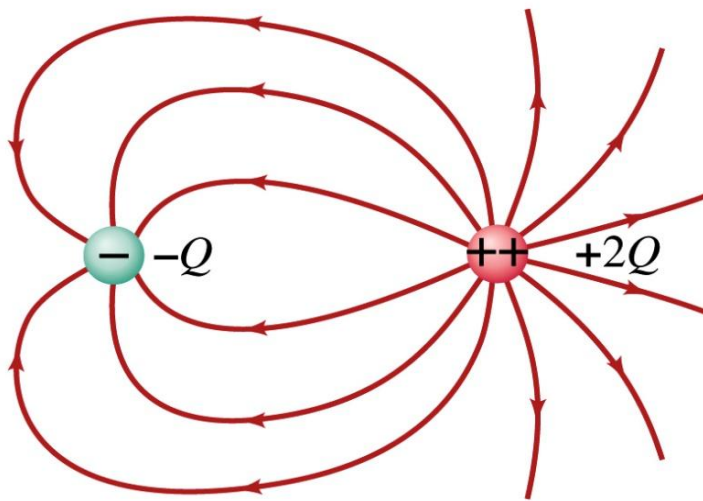
Non-uniform objects will have non-uniform fields. However the basic rules still apply with regard to direction and proximity of field lines.

Note that the electric field strength is greatest where the radius of curvature is smallest.

A pair of charges of opposite sign is called a dipole. Note that the shape of the electric field for a dipole is very similar to that of the magnetic field diagram for a bar magnet.



Electric field lines from repelling charges.



Electric field lines around a pair of charges.

Example 8

Two charges of magnitude -20 nC and $+80\text{ nC}$ are placed a distance 0.35 m apart. A particle is placed on a line that lies directly from one charge to the other at a distance of 0.15 m from the -10 nC charge. Calculate the magnitude and direction of the field strength at the -10 nC point.

Example 9

An object of mass 10 g and charge + 50 μC is suspended in mid-air by an electric field. Calculate the magnitude and direction of the electric field strength and electric force required in this situation.

Example 10

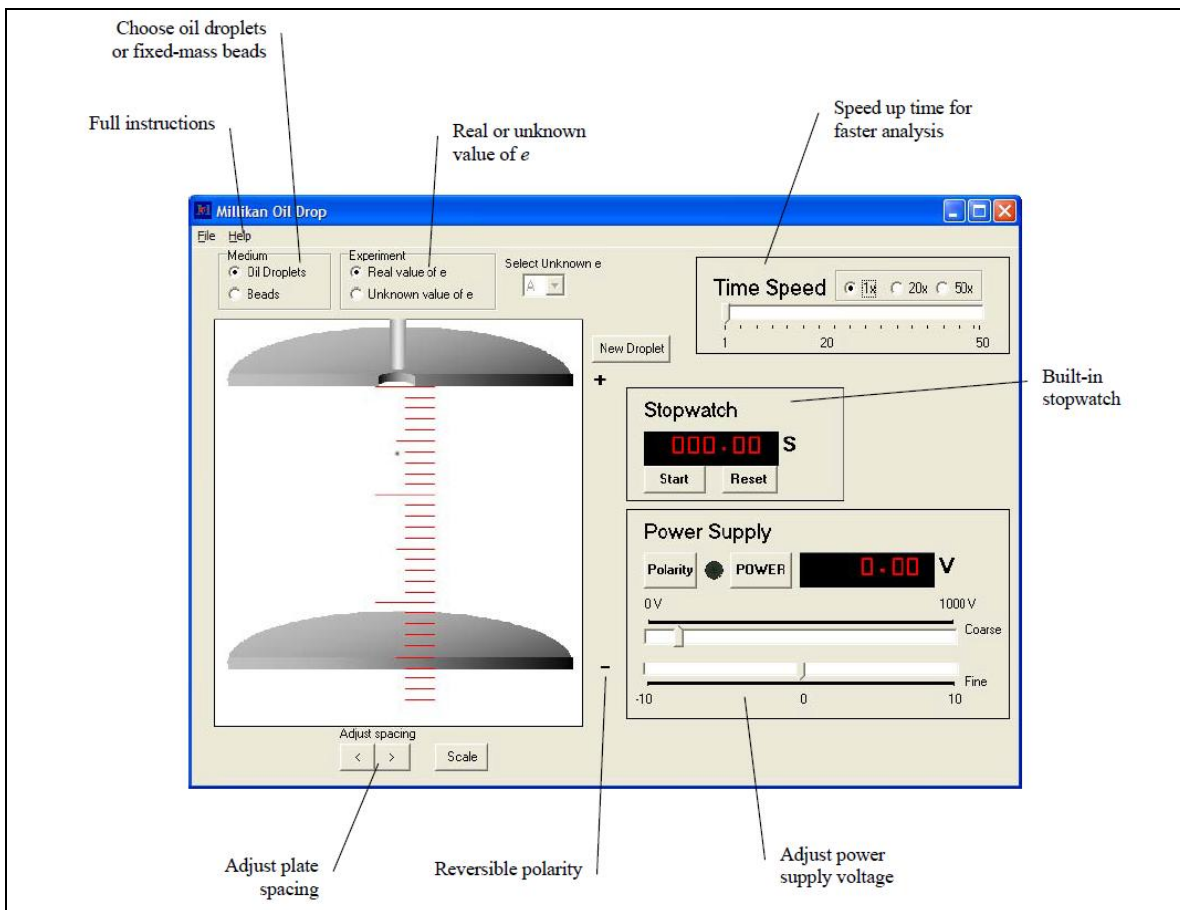
An electron is accelerated from rest by an electric field of strength 2.00 N/C. (a) Calculate the initial acceleration of the electron. (b) Calculate the time taken to accelerate the electron to 1.1×10^7 m/s. (c) Calculate the distance travelled by the electron during this acceleration.

LAB: Millikan's Oil Drop Simulation

<http://www.teachscience.net/2011/02/07/millikan-oil-drop-simulation/>

Aim

To use a computer simulation of Millikan's Oil Drop experiment to determine the charge on the electron.



Theory

If the mass of the oil drop is known (the fixed value of the oil bead in the simulation is $1 \times 10^{-15} \text{ kg}$), then when it becomes suspended in the E-field, its weight is balanced by the electrostatic attraction,

$$qE = mg$$

As $E = V/d$, we get:

$$q = \frac{mgd}{V}$$

So far, so easy. The problem in real life is that the mass of the droplet cannot be measured – it is far too small. However, it can be determined by the Stoke's Equation for a sphere falling through a fluid at terminal velocity, including compensating for the buoyant upthrust due to the density of the air, ρ_o :

$$r = \sqrt{\frac{9\eta v_o}{2g(\rho - \rho_o)}}$$

Assuming that that oil drop is a sphere, where:

$$m = \rho V = \frac{4}{3}\pi r^3 \rho$$

We get the rather cumbersome:

$$m = \frac{4}{3}\pi\rho\left(\sqrt{\frac{9\eta v_o}{2g(\rho - \rho_o)}}\right)^3$$

Once the density of the oil, the density of air and the viscosity of air are factored in, this simplifies down to:

$$m = 3.32477 \times 10^{-9} v_o^{3/2}$$

Part 1 – Using a bead of oil of a fixed mass (easier)

Set the oil drop to “bead”. This fixes the mass of the bead to 1×10^{-15} kg. Therefore we can simply suspend the bead between the plates to determine the charge on the bead. Turn the power supply to the plates on – the voltage can be adjusted by both a coarse and a fine tune. It is help to run the time faster (use the buttons to the upper right of the screen) to determine that the bead has been completely suspended. Use the equation in the theory section above to calculate the charge on the oil drop. Repeat this at least 10 times. Either use the known value of e to plot a scatter graph of number of electrons v charge or an analysis of the lowest common denominator to determine a value for e .

Part 2 – Using a bead of oil with an unknown mass (harder)

Set the oil drop to “droplet”. The main difference with this approach is that you will need to measure the terminal velocity of the drop with NO ELECTRIC FIELD before then turning on the field and suspending the SAME drop. This can be tricky and takes practice – which is why Millikan et al took nearly 20 years performing the real experiment!

The analysis requires that the mass of the droplet be determined so that the charge on the droplet can be calculated. Otherwise, the analysis to calculate the charge on the electron is the same as above.

Lab notes:

13.4 Electric Potential

Objectives:

- Understand the concept of electric potential and be able to draw equipotentials.
- To be able to calculate the electric potential due to a point charge.
- To be able to calculate the electric potential between two point charges.

Notes:

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The electric potential, V , is a property of the electric field, the electric potential energy of a charge within the field depends not only on the size of the charge but also the potential.

The electric potential is defined as being numerically equal to the work done in bringing a unit positive charge from infinity (where the potential is zero) to that point.

$$V = -\frac{kQ}{r}$$

It follows that:

Electric Potential Energy = Charge \times Electric Potential

$$E.P.E. = Q \times V$$

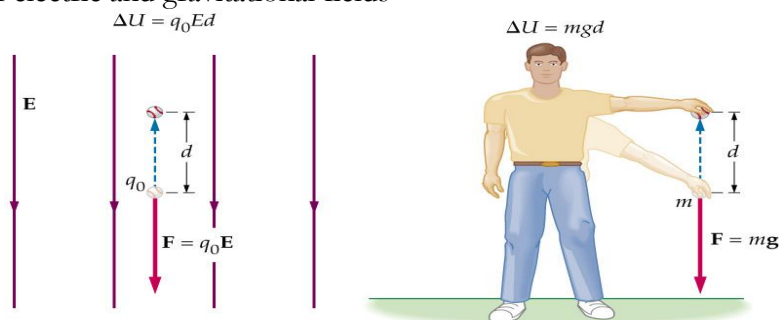
$$\Delta EPE = -Q E \Delta x$$

Electric potential is measured in Volts or Joules per coulomb. The relationship between field strength and potential is as follows:

$$E = -\frac{V}{r}$$

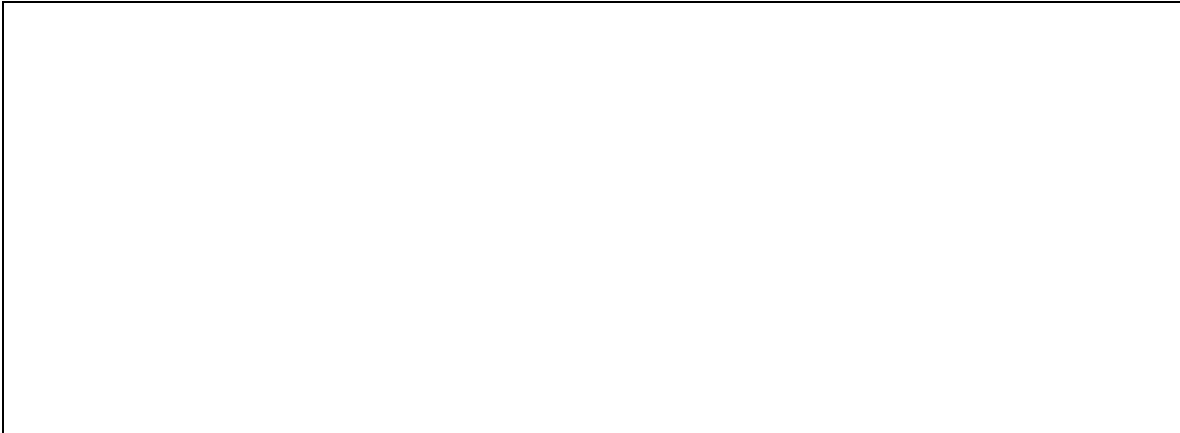
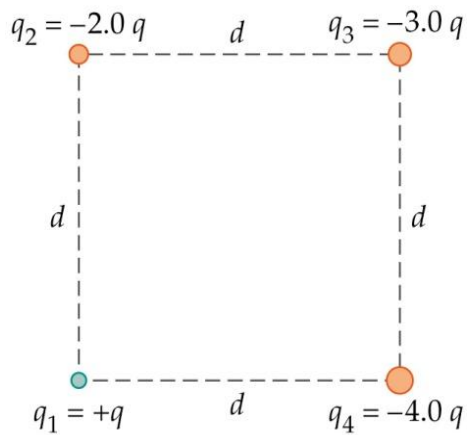
Once again the relationship between electric field strength and electric potential is easily seen. The potential is proportional to how quickly strength of the field changes. Where the potential is greatest the electric field strength is greatest and the lines of electric force are close together. Electric potential is the change in space of electric field strength; it is the gradient of electric field strength. Both electric potential and electric potential difference are scalar quantities.

Comparison of electric and gravitational fields



Example 11

A particle of mass 2g and charge $+10\text{ }\mu\text{C}$ moves a distance of 1.0m through an electric field of strength $+10\text{ N/C}$. If it moves parallel to a field line calculate the work done and change in electric potential energy of the charge. Calculate the final velocity of the charge if it started from rest.

**Example 12**

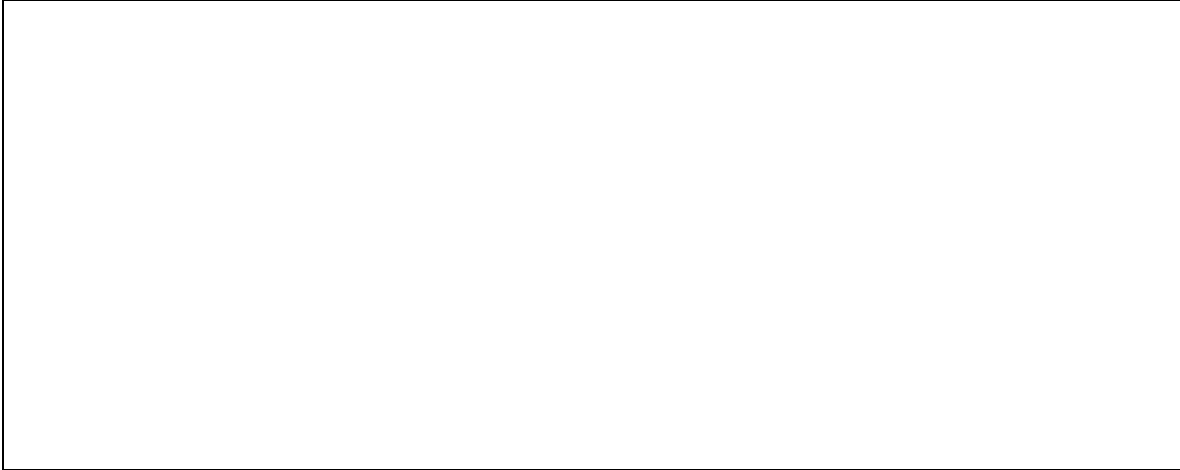
Calculate the electric potential at q_1 .

Find the work done in bringing the charge q_1 to its position from infinity if $q = 1\text{ }\mu\text{C}$ and $d = 5\text{ cm}$.

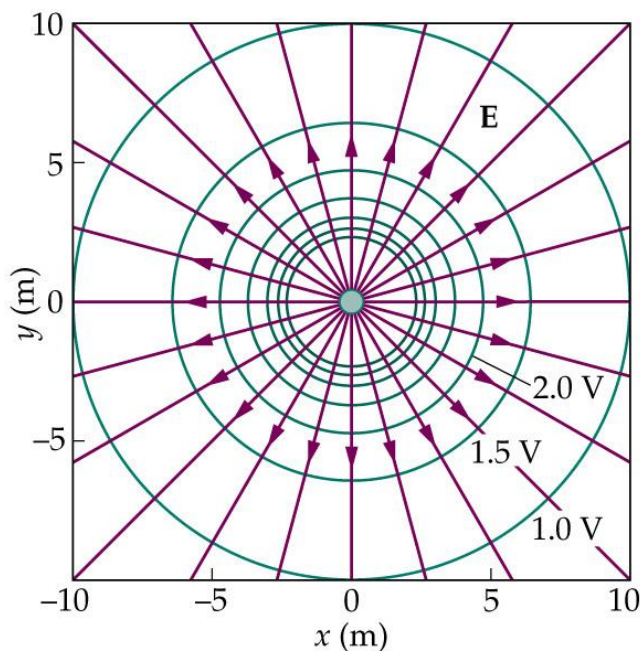


Example 13

A proton is released from rest in an electric field of strength 250 N/C . Calculate the change in potential energy when it has travelled 7.0 cm . Calculate the change in potential energy for an electron that travels the same path.

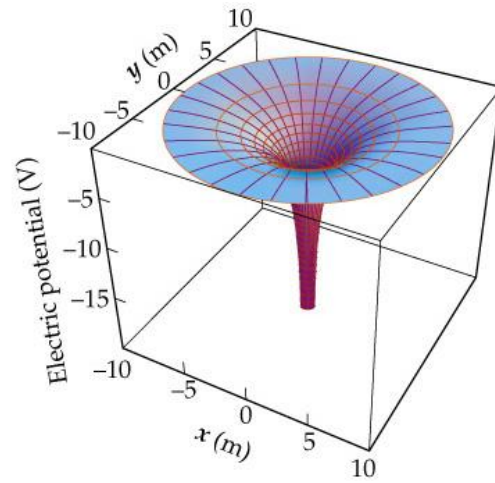
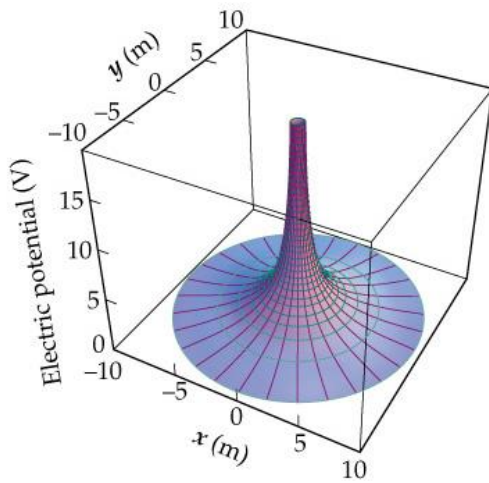
**Equipotentials**

We can draw diagrams joining points of the same electric potential. Further lines of equal potential and a constant difference in potential can be drawn. These diagrams aid our understanding of electric field.



This diagram shows both equipotentials (no arrows it is a scalar quantity) and lines of electric field strength (arrows from positive to negative).

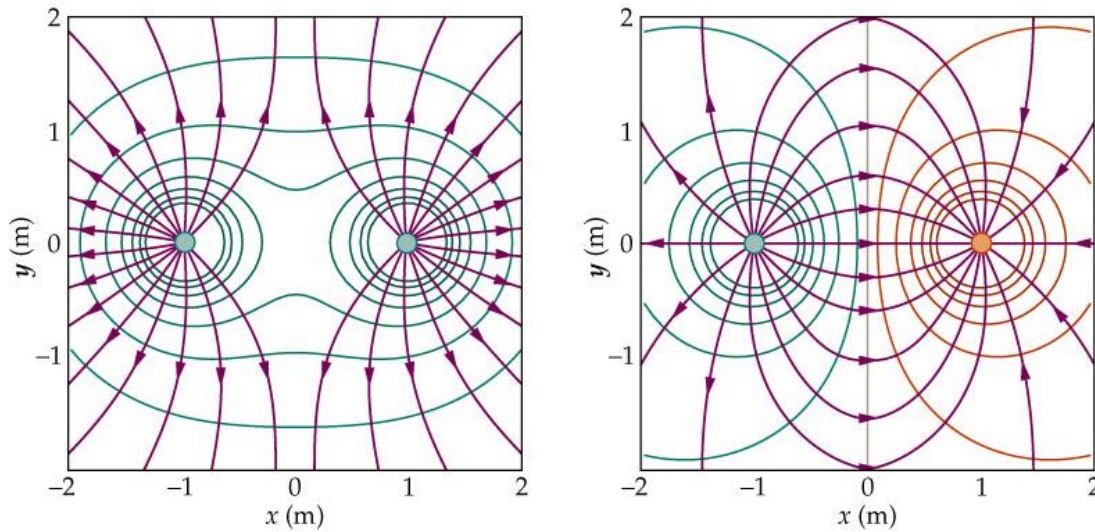
Equipotentials are at right angles to lines of electric field strength.



The three dimensional representation, shown above is for a positive charge (left) and negative charge (right).

Sketching a potential well and hill.

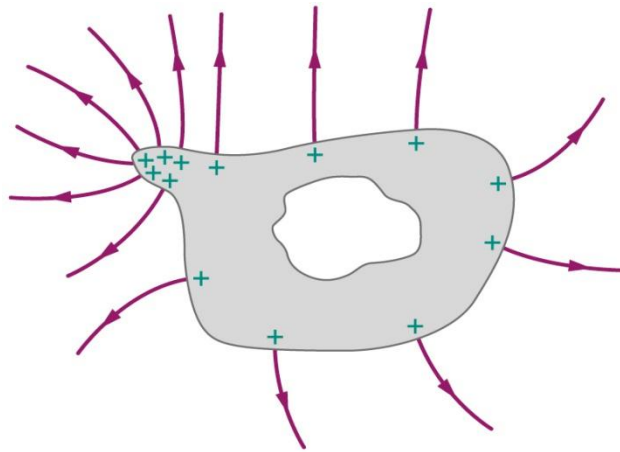
Electric force and electric field strength act in the same direction, whereas the electric potential is at right angles to the electric field. It follows that no work is done in moving along a line of equal potential as force and distance are at right angles to one another.



These diagrams display both lines of electric field strength and equipotential lines. On the left we have two positive charges, on the right a positive and negative charge.

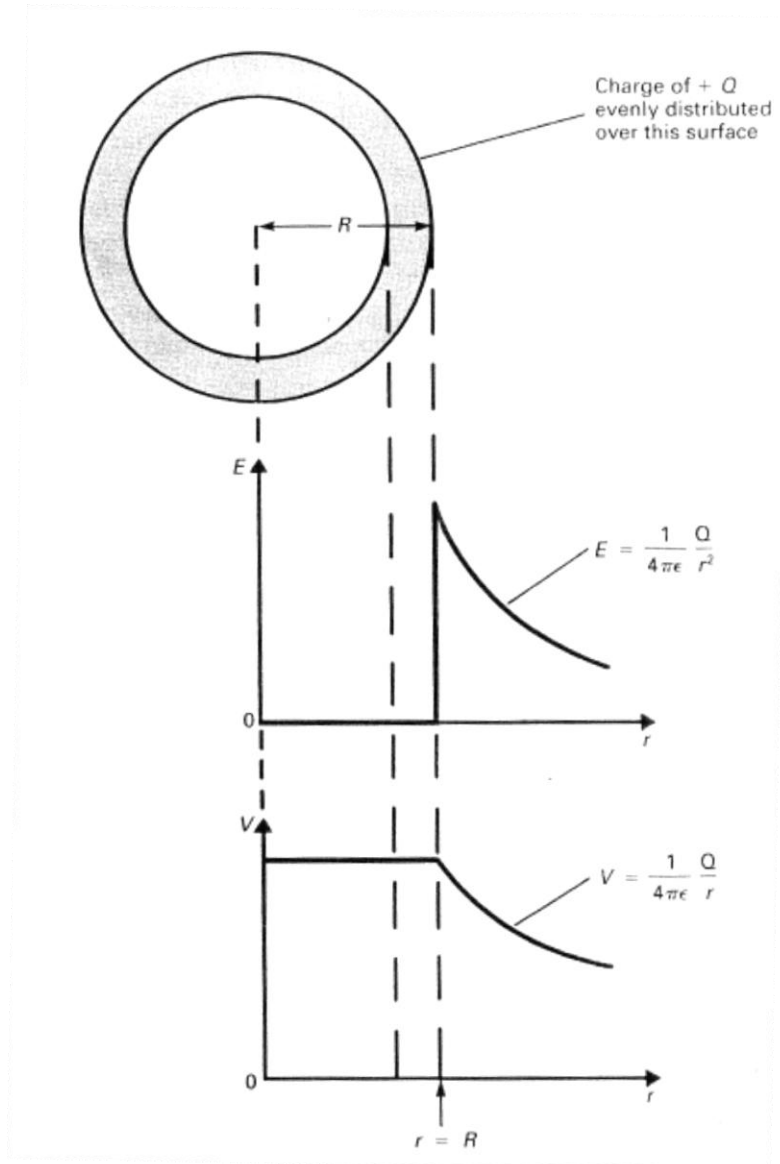
Charged Conductors

In any conductor there are charge carriers which are free to move; for a metal these take the form of loosely bound ‘free’ electrons. If a conductor is charged at a point its potential in that area will have changed. Unlike an insulator the charge carriers on a conductor will redistribute themselves until the electric potential is the same at all points. In equilibrium every part of the material of a conductor is at the same potential. This implies that the potential gradient is zero.



This requires that there is no charge within the material of the conductor; the charge must therefore reside on the surface of the conductor. Electrons will move until they minimise the forces acting upon themselves. The position of least force will be at the surface of the material; this is why charge will only be found at the surface of a conducting object. The electric field at the surface of a conductor must be at right angles to the surface to satisfy the above criteria as electric field lines are always at right angles to electric potential lines. Inside a solid or charged or hollow conductor the electric potential is constant, hence the electric field strength is zero.

A hollow conducting sphere with charge $+Q$ (*** LEARN!*****)**



13.5 Uniform Electric Fields

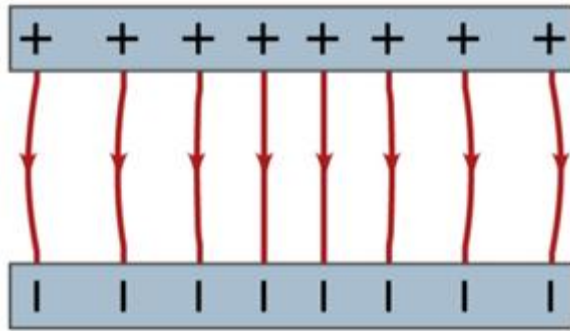
Objectives:

- Know how to calculate the electric field strength between two charged plates.
- To be able to describe an electron gun and to be able to calculate the speed of an accelerating electron.

Notes:

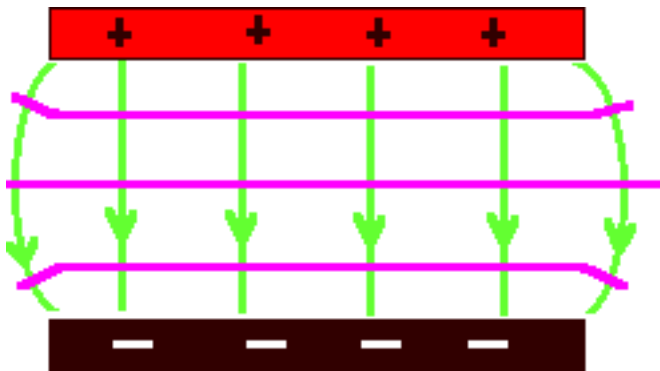
A pair of parallel conducting plates of equal cross sectional area is connected to a power supply so that one plate is charged positive and one negative.

Electric field strength between plates



Between a pair of long straight charged plates the lines of an electric field are approximately parallel.

Electric potential and field strength between plates



Lines of electric potential will be parallel to the plates and perpendicular to the electric field.

The potential difference between the positive and negative plates is V at all points along their surfaces. The electric field strength can be given by:

Electric Field Strength = Electric Potential Difference \div Distance between Plates

$$E = \frac{V}{d}$$

Example 14

Two parallel metal plates are held a distance of 10 cm apart. The upper plate has a potential of 12 V and the lower plate a potential of 4 V. (a) Calculate the potential and field strength midway between the two plates. (b) Calculate the potential and field strength 2 cm above

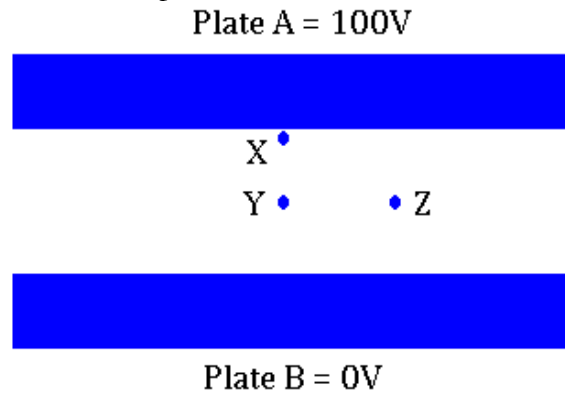
the lower plate. (c) A charge of magnitude $4\mu\text{C}$ is placed 2 cm above the lower plate, calculate its electric potential energy.

Example 15

Two parallel metal plates are held a distance of 20 cm apart. The upper plate has a potential of 200 V and the lower plate has a potential of -200 V. Calculate the field strength between the plates. If the voltage on the plates is doubled while the gap is reduced to 4 cm what will be the effect on the field strength.

Example 16

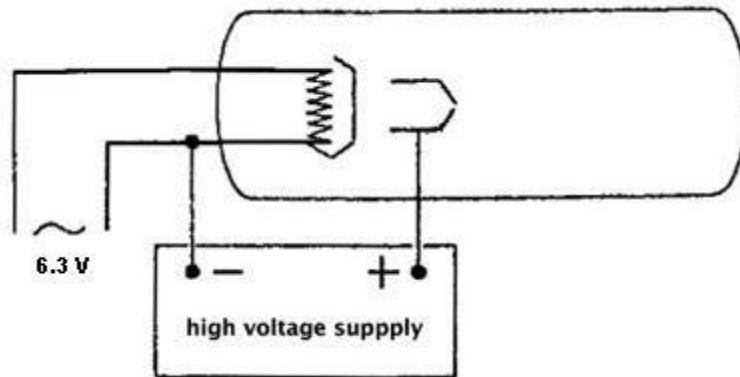
An electric field exists between two plates. The upper plate 'A' is set to 100 V, while the lower plate 'B' is set to 0V. The gap between the plates is 5cm and points X, Y and Z are as shown with Y midway between the plates.



- (a) What is the direction of the electric field?
- (b) What is the electric field strength at X?
- (c) What is the electric field strength at Y?
- (d) What is the work done in moving an electron from X to Y?
- (e) What is the work done in moving an electron from Y to Z?
- (f) If a charge of $2.0 \mu\text{C}$ were placed at Y, what force would be exerted on it?

Electron Gun

These are an extremely important use of electric fields. They are found in old style TVs and many types of particle accelerator. They work by releasing electrons by thermionic emission from a cathode and then using an electric field to accelerate them rapidly.



The low voltage ac current heats up the cathode, which causes the more energetic electrons to be “boiled off”. They are then accelerated by the high voltage induced electric field.

The KE of the electrons is found from the electric field, which is related to the accelerating voltage, V .

$$KE = \text{charge} \times \text{voltage}$$

$$KE = eV = \frac{1}{2} mv^2$$

Therefore the velocity of the electron as it leaves the gun is:

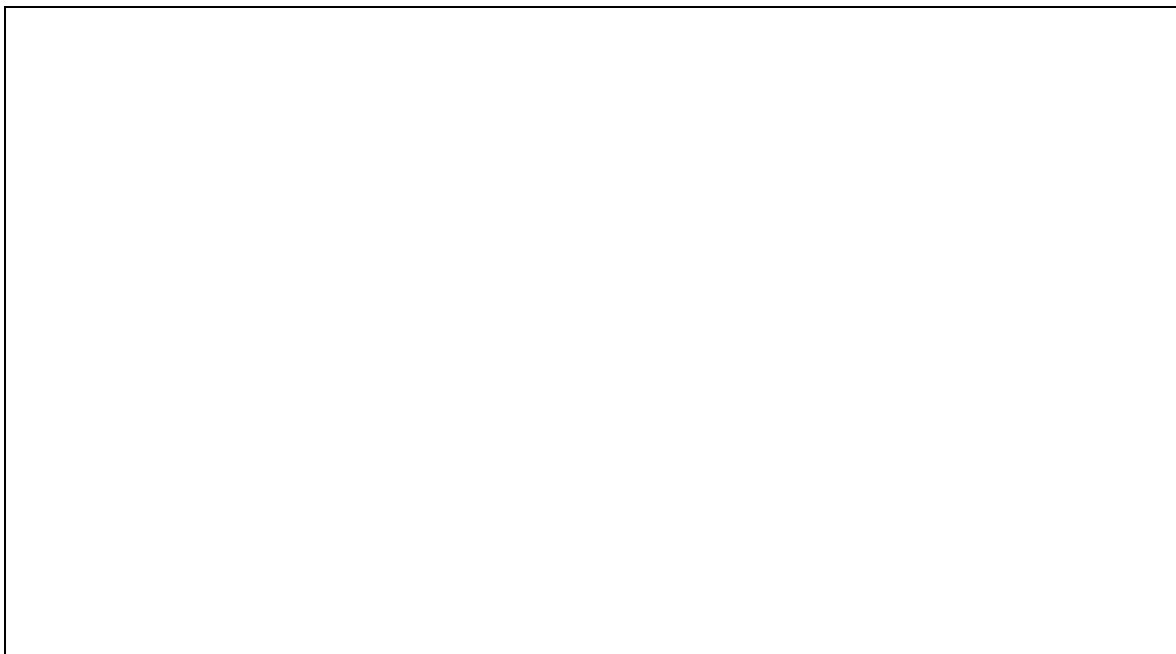
LAB: The Path of an Electron in a Uniform Electric Field

Aim

To use the equations of electrostatics and mechanics to describe the path of an electron as it traverses an uniform electric field between two high voltage plates.

Theory

There is a lot of varied physics involved here, and as such this becomes an ideal AP Free Response question. The electron is accelerated up to a high speed by an electron gun mounted inside a low pressure teletron tube. It then passes between two metal plates that are charged to a high voltage by the same external 5kV power supply. (in an ideal world the HT supplies would be different, however, few schools have more than one unit!) Behind the plates is mounted a phosphorous screen that illuminates as the electrons pass across it. The path of the electrons can clearly be seen and the parameters measured.



Firstly, we need to calculate the speed of the electron as it is accelerated by the gun. It is rightly assumed that the electron does not achieve relativistic speeds.

$$KE = \frac{1}{2}mv^2 = eV$$

Once between the plates, the electron is subject to a vertical force due to the electrical field. The force due to gravity can safely be ignored as it is close to zero.

$$F = qE = \frac{eV}{d}$$

Where V is the voltage across the plates and that which accelerates the electron, and d is the separation between the plates. This vertical force accelerates the electron downwards (or upwards depending on the polarity of the plates), while the electron moves at a constant horizontal velocity. This results in a parabolic trajectory, much like a ball thrown horizontally in a gravity field. The mechanics are the same. The next step is to use this force to calculate the vertical acceleration and hence the time of flight for the set vertical drop of y .

$$a = \frac{F}{m} = \frac{eV}{md}$$

From

$$x = v_o t + \frac{1}{2} a t^2$$

And $v_o = 0$ in the vertical direction:

$$t = \sqrt{\frac{2ymd}{eV}}$$

Assuming that the deflection voltage and the accelerating voltage are equal, this yields a horizontal range of:

$$x = \sqrt{4yd}$$

Question: Show this final step!

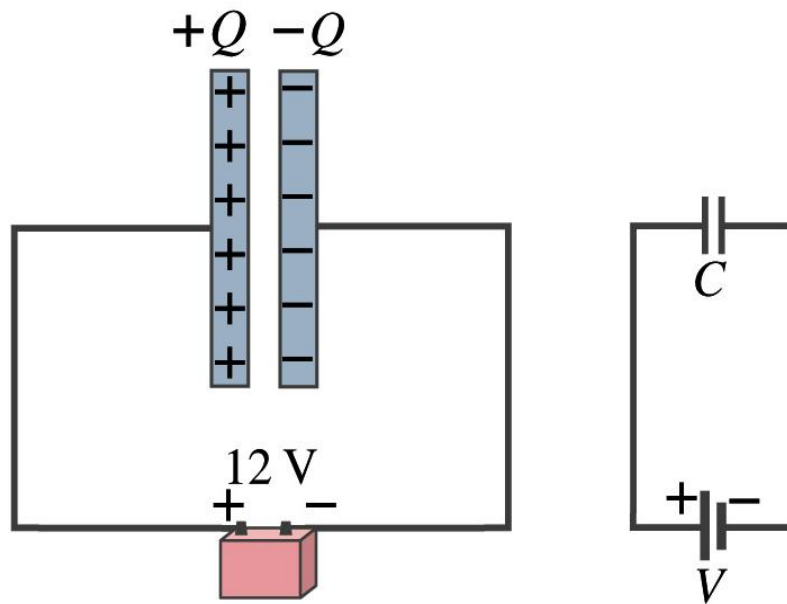
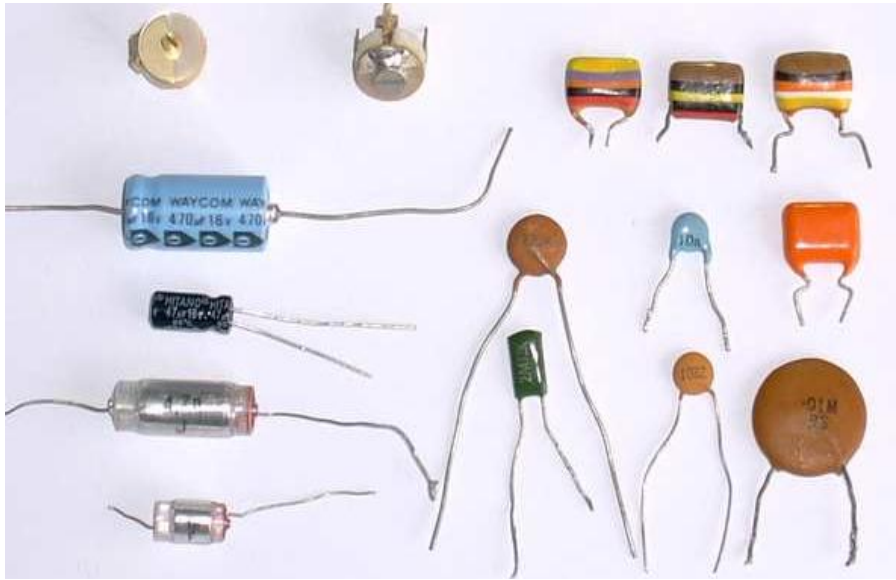
Lab notes:

13.6 Capacitance

Objectives:

- Know how two parallel plates can store charge as a capacitor.
- To know and be able to use the equation for the capacitance of a capacitor.
- To be able to calculate the energy stored by a capacitor

Notes:



When the cell is connected in the simple circuit shown above there is a momentary electric current. Initially electrons flow from the positive plate through the battery and round onto the negative plate. As the charge flows from one to the other a net positive charge occurs at one plate and a net negative charge at the other. This difference of electric charge creates an electric field between the plates. The electric field, and hence potential difference, between the two plates opposes the field produced by the battery. The force on the electrons still moving in the circuit is the resultant of these two fields. As the overall field acting on the electrons within the circuit decreases the current decreases. The plates themselves will only become fully charged when time of charging is infinite. When the plates are discharged the process repeats itself with a high initial current decreasing as the charge on the plates decreases.

Charge stored on a Capacitor

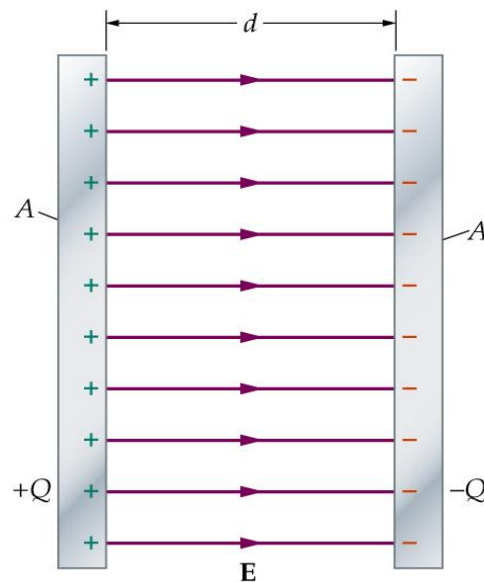
Charge on plates = Electric potential between plates \times Capacitance of plates

Charge = Voltage \times Capacitance

$$Q = \Delta V \times C$$

The capacitance of a capacitor is simply the ratio of the amount of charge on each plate divided by the voltage between the plates. Capacitance is measured in coulombs per volt or farads. A farad is very large unit, practical capacitance's are normally of the order of nano (n) or micro (μ) farads. Q is used for charge as it was originally described as 'the quantity of electricity'. Compare this with the symbol Q that is used for heat in the first law of thermodynamics, here again it was known as 'the quantity of heat supplied'.

Practical Capacitors



The relationship between the quantities in the diagram can be shown to be:

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Where:

C = Capacitance (F).

ϵ_0 = Permittivity of free space (F m^{-1}).

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}.$$

ϵ_r = Relative permittivity (no unit).

A = Cross sectional area of overlapping plates (m^2).

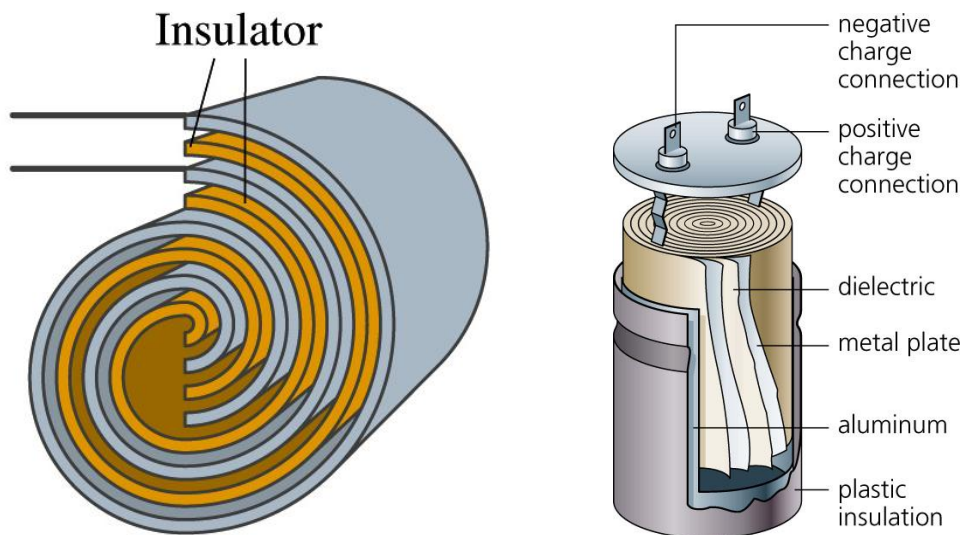
d = Distance between parallel plates (m).

The relative permittivity of the dielectric material between the plates can vary upwards from a value of one for a vacuum.

Material	Relative Permittivity
Air	1.000576
Polythene	2.3
Glass	5.0
Water	80

The atoms and molecules within a dielectric, such as water, are often polarised. Initially their orientation within the structure of the dielectric is random. When a charge builds up on the plates of the capacitor the molecules in the dielectric experience a torque. The forces causing rearrangement within the structure oppose the electric field between the plates. The dielectric allows the plates to store more charge and hence more energy between the two plates.

From the formula we can clearly see that a high capacitance results from a large surface area, a small separation and a high permittivity. Capacitors need to be relatively cheap and reliable as well as very small.

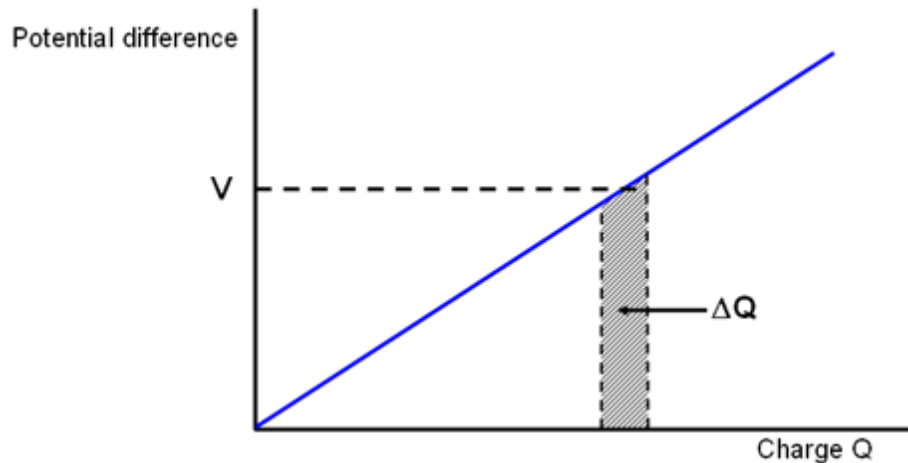


The metal foil above acts as our parallel plates with the thin strips of paper as our dielectric. The four sheets can be rolled into a cylinder of a small volume achieving the small size we require. The foil strips extend beyond the paper to make contact with its protective casing.

Energy and Capacitance

When electrons move from one plate to another around an electrical circuit, work must be done. Part of this work done manifests itself as the energy stored on the plates of the capacitor. This stored energy can be released or transferred when the capacitor is discharged.

The work done in moving a single charge q between two plates with a potential difference V is given by $E = qV$. For a charging capacitor the voltage between the two plates changes as the charge is transferred.



The work done in moving charge from one plate to another is equal to the energy stored by the capacitor and can be calculated from the graph above. It can be shown that:

$$E = \frac{1}{2} Q \times V$$

$$E = \frac{1}{2} C \times V^2$$

Example 17

Calculate the charge stored on a $1000 \mu\text{F}$ capacitor charged to 10 volts.

Example 18

Calculate the energy stored by a $1000\ \mu\text{F}$ capacitor charged to 10 volts.

Example 19

If the plates of a capacitor are doubled in area whilst their separation halves, what will be the new capacitance?

Example 20

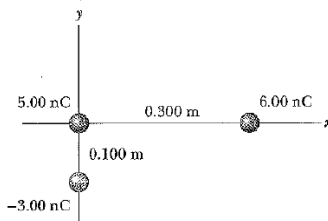
A parallel plate capacitor has an area of $5.00\ \text{cm}^2$, and the plates are separated by $1.00\ \text{mm}$ with air between them. The capacitor stores a charge of $400\ \text{pC}$. (a) What is the potential difference across the plates of the capacitor? (b) What is the magnitude of the uniform electric field in the region between the plates?

Example 21

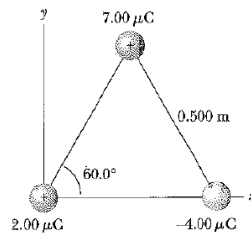
A fully charged defibrillator contains 1.20 kJ of energy stored in a 1.10×10^{-4} F capacitor. In a discharge through a patient, 6.00×10^2 J of electrical energy is delivered in 2.50 ms. (a) Find the voltage needed to store 1.20 kJ in the unit. (b) Find the average power delivered to the patient.

Assignment 13.1 - Electric Force

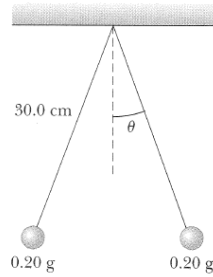
1. Calculate the force between two charges of magnitude 4.0 nC and -2.0 nC, when they are a distance of 10 cm apart.
2. Two point charges, A and B, experience an attractive force of 2.16×10^{-6} N at a particular distance. If the charge of A is 3 nC and the charge of B is 5 nC, calculate the distance between them. What do you know about the sign of the charges A and B?
3. Question 1, page 524. A charge of 4.5×10^{-9} C is located 3.2 m from a charge of -2.8×10^{-9} C. Find the electrostatic force exerted by one charge on the other.
4. Imagine if a quantity of 1g of protons were placed on one side of the Earth and 1 g of electrons on the other. If the approximate radius of the Earth is 6400 km, calculate the force of attraction between the two masses.
5. Question 3, page 525. An alpha particle is sent at high speed toward a gold nucleus (79 protons). What is the electrical force acting of the alpha particle when it is 2.0×10^{-14} m from the gold nucleus?
6. Question 5, page 525. The nucleus of ^8Be , which consists of 4 protons and 4 neutrons, is very unstable and spontaneously breaks into two alpha particles (each consisting of two protons and two neutrons). (a) What is the force between the two alpha particles when they are 5.00×10^{-15} m apart, and (b) what will be the magnitude of the acceleration of the alpha particles due to this force? Note that the mass of an alpha particle is 4.0026 u.
7. Question 6, page 525. A molecule of DNA is 2.17 μm long. The ends of the molecule become singly ionised – negative on one end and positive on the other. The helical molecule acts like a spring and compresses 1.00 % upon becoming charged. Determine the effective spring constant of the molecule.
8. Question 9, page 525. Two identical conducting spheres are placed with their centres 0.30 m apart. One is given a charge of 12×10^{-9} C the other is given -18×10^{-9} C. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. Find the electrostatic force between the two after equilibrium has been reached.
9. Question 11, page 525. Three charges are arranged as shown in the figure below. Find the magnitude and direction of the electrostatic force on the charge at the origin.



10. Question 13, page 525. Three point charges are located at the corners of an equilateral triangle as in figure 15.13. Calculate the net electric force on the 7.00 nC charge.



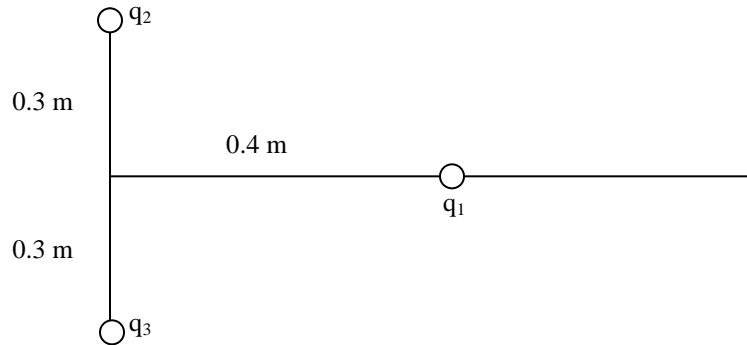
11. Question 15, page 526. Two small metallic spheres, each of mass 0.20 g , are suspended as pendulums by light strings from a common point as shown in figure 15.15. The spheres are given the same electric charge, and it is found that they come to equilibrium when each string is at an angle of 5.0° with the vertical. If each string is 30.0 cm long, what is the magnitude of the charge on each sphere?



Assignment 13.2 - Electric Field Strength and Coulombs Law

Electric force questions

1. For the figure below, $q_1 = -1.0 \mu\text{C}$, $q_2 = 2.0 \mu\text{C}$ and $q_3 = -2.0 \mu\text{C}$. Find the magnitude and direction of the total force on charge q_1 .



2. Two charges of $10 \mu\text{C}$ and $-5 \mu\text{C}$ are 9 cm apart. Where must a third charge be placed in order that the resultant force acting upon it will be zero. (Note: you do not need to solve for the magnitude of the third charge, solve for distance only). (hard question)

Electric field strength questions

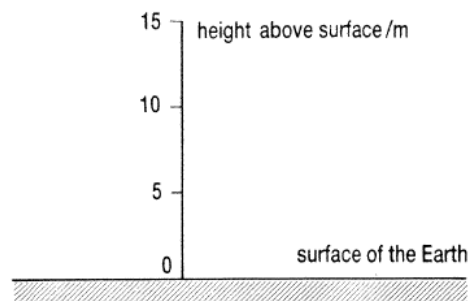
3. A charge of magnitude $+20 \mu\text{C}$ is placed upon a surface. Find the electric field strength at a distance of 0.1 m and a distance of 0.1 mm.
4. Two charges of magnitude $+10 \text{ nC}$ and $+60 \text{ nC}$ are placed a distance 0.25 m apart. A particle is placed on a line that lies directly from one charge to the other at a distance of 0.05 m from the $+10 \text{ nC}$ charge. Calculate the magnitude and direction of the field strength at this point.
5. Two charges of magnitude $+25 \mu\text{C}$ and $-40 \mu\text{C}$ are placed on a surface at a distance of 3.0 m from each other. Find the point, apart from infinity, where the resultant electric field strength is zero on the line joining the two charges.
6. Question 17, page 526. An object with a net charge of $24 \mu\text{C}$ is placed in a uniform electric field of 610 N/C , directed vertically. What is the mass of the object if it “floats” in the electric field. (like question 2)
7. Question 21, page 526. A Styrofoam ball covered with a conducting paint has a mass of $5.0 \times 10^{-3} \text{ kg}$ and has a charge of $4.0 \mu\text{C}$. What electric field directed upward will produce an electric force on the ball that will balance its weight?
8. Question 23, page 526. A proton accelerates from rest in a uniform electric field of 640 N/C . At some later time, its speed is $1.20 \times 10^6 \text{ m/s}$. (a) Find the magnitude of the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it move in that time interval? (d) What is the kinetic energy at the later time?

Assignment 13.3 - Electric fields 1

1. For each of the following, state whether it is a scalar or a vector and give an appropriate unit:
 - i) electric potential,
 - ii) electric field strength.
2. In a sodium chloride crystal a sodium ion has a charge of $+ 1.6 \times 10^{-19} \text{ C}$ and a chlorine ion has a charge of $- 1.6 \times 10^{-19} \text{ C}$. They are found by X-ray diffraction to be a distance of 0.2 nm apart. What force exists between them if they are in a vacuum? ($\epsilon_0 = 8.9 \times 10^{-12} \text{ F m}^{-1}$)
3. A conducting sphere of radius 5.0 cm has a charge of $4.0 \times 10^{-6} \text{ C}$. Find the potential:
 - i) 6.0 cm from the centre of the sphere.
 - ii) 5.0 cm from the centre of the sphere.
 - iii) 4.0 cm from the centre of the sphere.
4. Points A and B are 0.10 m apart. A point charge of $+ 3.0 \times 10^{-9} \text{ C}$ is placed at A and a point charge of $- 1.0 \times 10^{-9} \text{ C}$ is placed at B.
 - i) X is the point on the straight line through A and B, between A and B, where the electric potential is zero. Calculate the distance AX.
 - ii) Show on a diagram the appropriate position of a point, Y, on the straight line through A and B where the electric field strength is zero. Explain your reasoning, but no calculation is expected.
5. Two light, conducting spheres, each of 6 mm diameter and having mass 10 mg, are suspended from the same point by fine insulating fibres 50 cm long. Due to electrostatic repulsion, the spheres are in equilibrium when their centres are 3 cm apart. What is:
 - i) the force of repulsion between the spheres,
 - ii) the charge on each sphere,
 - iii) the potential of each sphere? (Assume $g = 10 \text{ m s}^{-2}$)
6. A large, hollow, metal sphere is charged positively and insulated from its surroundings. Sketch graphs of (i) the electric field strength, and (ii) the electric potential, from the centre of the sphere to a distance of several diameters.

Assignment 13.6 - Electric fields 2

1. Find the velocity of an electron if it falls through a potential difference of 1 V, and its charge is -1.60×10^{-19} C.
2. A small charge of $+6.2$ nC is moved a distance of 2.0 mm from one conducting plate to another against a field of $20\,000$ N C $^{-1}$. Find the work done on the charge and the potential difference between the plates.
3. An electron in the vacuum of a cathode ray tube moves from rest at a point where the potential is -1400 V to a point where the potential is zero. Find:
 - i) the gain in potential of the electron.
 - ii) the loss in potential energy of the electron (why are these of opposite sign?).
 - iii) the gain in kinetic energy of the electron.
 - iv) the final speed of the electron.
4. A beam of electrons travelling at 1.3×10^7 m s $^{-1}$ enters a uniform electric field between two plates of length l , separated by a distance d . One plate is at a potential of $+50$ V and the other is at -50 V. If $l = 0.060$ m and $d = 0.020$ m, find θ , the angular deflection of the beam. Hint: consider the velocity components of motion parallel and perpendicular to the plates.
5. The diagram below shows a small area of the surface of the Earth, assumed to be flat and a conductor of electricity. There is a uniform electric field at the surface of 300 V m $^{-1}$ directed away from the surface.
 - i) On a similar sketch, draw three field lines and three equipotentials. Label the equipotentials with their potential relative to the Earth.
 - ii) Calculate the work done in moving a charge of $+5.0 \times 10^{-6}$ C;
 - a) a distance of 4.5 m parallel to the Earth's surface when it is 2.0 m above it, and
 - b) from a height of 2.0 m down to the surface of the Earth.

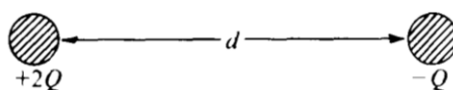


Assignment 13.7 - Capacitors

1. Calculate the charge stored by a $20\ \mu\text{F}$ capacitor charged to $300\ \text{V}$.
2. Calculate the potential difference required to give a $20\ \mu\text{F}$ capacitor a charge of $1.0\ \text{mC}$.
3. Calculate the capacitance of a capacitor which has a charge of $0.3\ \text{mC}$ when charged to a potential difference of $500\ \text{V}$.
4. A capacitor stores a charge of $30\ \text{mC}$ when connected to a battery of potential difference of $150\ \text{V}$. What is its capacitance?
5. What charge would there be on the plates if the capacitor (from the previous question) were connected to a potential difference of $250\ \text{V}$?
6. How much energy is stored in a capacitor which stores $30\ \text{mC}$ of charge when a potential difference of $150\ \text{V}$ is applied across it.
7. How much energy is stored by a capacitor of capacitance $20\ \mu\text{F}$ when a potential difference of $300\ \text{V}$ is applied across it.
8. A capacitor of $10.0\ \mu\text{F}$ is connected momentarily to a $400\ \text{V}$ d.c. source. How much charge is stored on the plates, and what is the energy of the system?
9. Question 22, page 564. (a) How much charge is on each plate of a $4.00\ \mu\text{F}$ capacitor when it is connected to a $12.0\ \text{V}$ battery? (b) If this same capacitor is connected to a $1.50\ \text{V}$ battery, what charge is stored?
10. Question 23, page 564. Consider the Earth and a cloud, the latter $800\ \text{m}$ above the planet, to be the plates of a parallel plate capacitor. (a) If the cloud layer has an area of $1.0\ \text{km}^2$, what is the capacitance? (b) If an electric field strength greater than $3.0 \times 10^6\ \text{N/C}$ causes the air to break down and conduct charge (lightning), what is the maximum charge the cloud can hold?
11. Question 24, page 564. The potential difference between a pair of oppositely charged parallel plates is $400\ \text{V}$. (a) If the spacing between the plates is doubled without altering the charge on the plates, what is the new potential difference between the plates? (b) If the plate spacing is doubled, from its original value, while the potential difference between the plates is kept constant, what is the ratio of the final charge on one of the plates to the original charge?
12. Question 25, page 564. An air filled capacitor consists of two parallel plates, each with an area of $7.60\ \text{cm}^2$ and separated by a distance of $1.80\ \text{mm}$. If a $20.0\ \text{V}$ potential difference is applied to these plates, calculate (a) the electric field strength between the plates, (b) the capacitance, and (c) the charge on each plate.

PAST AP QUESTIONS

1. A solid conducting sphere is given a positive charge Q . How is the charge Q distributed in or on the sphere?
- (A) It is concentrated at the center of the sphere.
 (B) It is uniformly distributed throughout the sphere.
 (C) Its density increases radially outward from the center.
 (D) It is uniformly distributed on the surface of the sphere only.



2. Two identical conducting spheres are charged to $+2Q$ and $-Q$, respectively, and are separated by a distance d (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is F_1 . After the two spheres are made to touch and then are re-separated by distance d , the magnitude of the force on the left sphere is F_2 . Which of the following relationships is correct?

(A) $2F_1 = F_2$ (B) $F_1 = F_2$ (C) $F_1 = 2F_2$ (D) $F_1 = 8F_2$

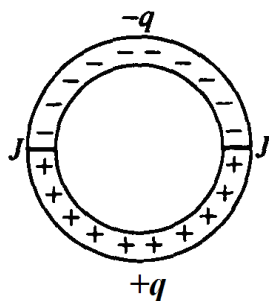
3. Two isolated charges, $+q$ and $-2q$, are 2 centimeters apart. If F is the magnitude of the force acting on charge $-2q$, what are the magnitude and direction of the force acting on charge $+q$?

<u>Magnitude</u>	<u>Direction</u>
(A) $2F$	Away from charge $-2q$
(B) F	Toward charge $-2q$
(C) F	Away from charge $-2q$
(D) $2F$	Toward charge $-2q$

4. **Multiple correct:**

Forces between two objects which are inversely proportional to the square of the distance between the objects include which of the following? Select two answers:

- (A) Gravitational force between two celestial bodies
 (B) Electrostatic force between two electrons
 (C) Nuclear force between two neutrons
 (D) Magnetic force between two magnets
5. Two small spheres have equal charges q and are separated by a distance d . The force exerted on each sphere by the other has magnitude F . If the charge on each sphere is doubled and d is halved, the force on each sphere has magnitude
- (A) F (B) $2F$ (C) $8F$ (D) $16F$

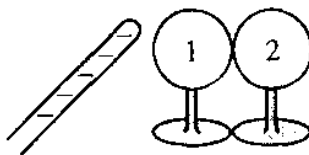


6. A circular ring made of an insulating material is cut in half. One half is given a charge $-q$ uniformly distributed along its arc. The other half is given a charge $+q$ also uniformly distributed along its arc. The two halves are then rejoined with insulation at the junctions J, as shown above. If there is no change in the charge distributions, what is the direction of the net electrostatic force on an electron located at the center of the circle?

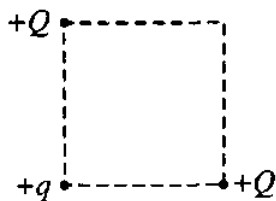
- (A) Toward the top of the page
 (B) Toward the bottom of the page
 (C) To the right
 (D) To the left

7. Two metal spheres that are initially uncharged are mounted on insulating stands, as shown above. A negatively charged rubber rod is brought close to, but does not make contact with, sphere X. Sphere Y is then brought close to X on the side opposite to the rubber rod. Y is allowed to touch X and then is removed some distance away. The rubber rod is then moved far away from X and Y. What are the final charges on the spheres?

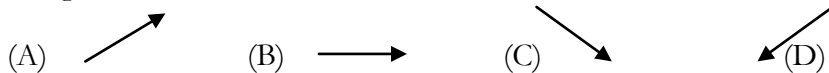
<u>Sphere X</u>	<u>Sphere Y</u>
A) Negative	Negative
B) Negative	Positive
C) Positive	Negative
D) Positive	Positive



8. Two initially uncharged conductors, 1 and 2, are mounted on insulating stands and are in contact, as shown above. A negatively charged rod is brought near but does not touch them. With the rod held in place, conductor 2 is moved to the right by pushing its stand, so that the conductors are separated. Which of the following is now true of conductor 2?
- (A) It is uncharged. (B) It is positively charged. (C) It is negatively charged.
 (D) It is charged, but its sign cannot be predicted.

Questions 9-10

9. As shown above, two particles, each of charge $+Q$, are fixed at opposite corners of a square that lies in the plane of the page. A positive test charge $+q$ is placed at a third corner. What is the direction of the force on the test charge due to the two other charges?

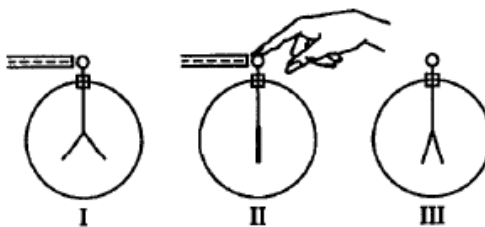


10. If F is the magnitude of the force on the test charge due to only one of the other charges, what is the magnitude of the net force acting on the test charge due to both of these charges?

(A) $\frac{F}{\sqrt{2}}$ (B) F (C) $\sqrt{2}F$ (D) 2

11. Suppose that an electron (charge $-e$) could orbit a proton (charge $+e$) in a circular orbit of constant radius R . Assuming that the proton is stationary and only electrostatic forces act on the particles, which of the following represents the kinetic energy of the two-particle system?

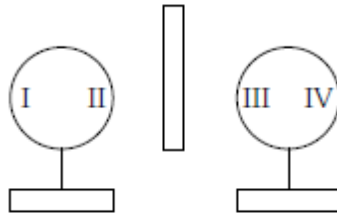
(A) $\frac{1}{8\pi\epsilon_0} \frac{e^2}{R}$ (B) $-\frac{1}{8\pi\epsilon_0} \frac{e^2}{R}$ (C) $\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2}$ (D) $-\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2}$



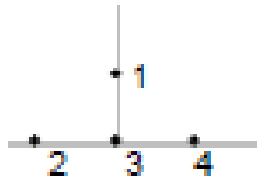
12. When a negatively charged rod is brought near, but does not touch, the initially uncharged electroscope shown above, the leaves spring apart (I). When the electroscope is then touched with a finger, the leaves collapse (II). When next the finger and finally the rod are removed, the leaves spring apart a second time (III). The charge on the leaves is

- (A) positive in both I and III
 (B) negative in both I and III
 (C) positive in I, negative in III
 (D) negative in I, positive in III

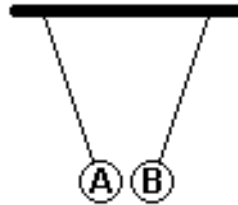
13. **Multiple Correct.** A positively charged conductor attracts a second object. Which of the following statements *could* be true? Select two answers
- (A) The second object is a conductor with positive net charge.
 - (B) The second object is a conductor with zero net charge.
 - (C) The second object is an insulator with zero net charge.
 - (D) The second object is an insulator with positive net charge.
14. Two positive point charges repel each other with force 0.36 N when their separation is 1.5 m. What force do they exert on each other when their separation is 1.0 m?
 (A) 0.81 N (B) 0.36 N (C) 0.24 N (D) 0.16 N
15. A point charge $+q$ is placed midway between two point charges $+3q$ and $-q$ separated by a distance $2d$. If Coulomb's constant is k , the magnitude of the force on the charge $+q$ is:
 (A) $2\frac{kq^2}{d^2}$ (B) $4\frac{kq^2}{d^2}$ (C) $6\frac{kq^2}{d^2}$ (D) $9\frac{kq^2}{d^2}$



16. A charged rod is placed between two insulated conducting spheres as shown. The spheres have no net charge. Region II has the same polarity as Region
 (A) I only (B) III only (C) IV only (D) I & IV only
17. When two charged point-like objects are separated by a distance R , the force between them is F . If the distance between them is quadrupled, the force between them is
 (A) $16 F$ (B) $4 F$ (C) $F/4$ (D) $F/16$
18. An electroscope is given a positive charge, causing its foil leaves to separate. When an object is brought near the top plate of the electroscope, the foils separate even further. We could conclude
 (A) that the object is positively charged.
 (B) that the object is electrically neutral.
 (C) that the object is negatively charged.
 (D) only that the object is charged.



19. Four positive point charges are arranged as shown in the accompanying diagram. The force between charges 1 and 3 is 6.0 N; the force between charges 2 and 3 is 5.0 N; and the force between charges 3 and 4 is 3.0 N. The magnitude of the total force on charge 3 is most nearly
 (A) 6.3 N (B) 8.0 N (C) 10 N (D) 11 N (E) 14 N

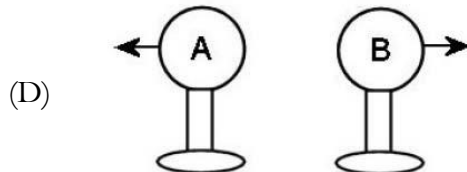
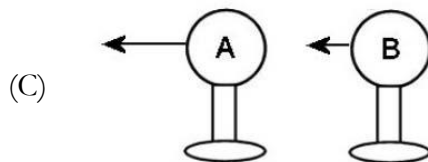
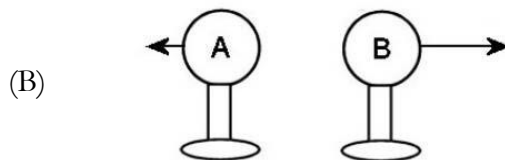
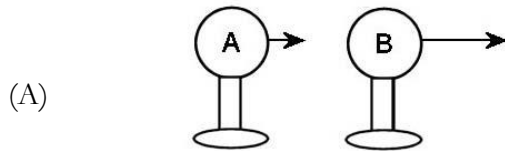


20. Two small hollow metal spheres hung on insulating threads attract one another as shown. It is known that a positively charged rod will attract ball A.
 Which of the statements can be correctly concluded about the charge on the balls?
 (A) Ball A has a positive charge
 (B) Ball B has a negative charge
 (C) Ball A and Ball B have opposite charges
 (D) Ball A is neutral
21. Two identical electrical point charges Q , separated by a distance d produce an electrical force of F on one another. If the distance is decreased to a distance of $0.40d$, what is the strength of the resulting force?
 (A) $6.3F$ (B) $2.5F$ (C) $0.40F$ (D) $0.16F$
22. A positively charged object is brought near but not in contact with the top of an uncharged gold leaf electroscope. The experimenter then briefly touches the electroscope with a finger. The finger is removed, followed by the removal of the positively charged object. What happens to the leaves of the electroscope when a negative charge is now brought near but not in contact with the top of the electroscope?
 (A) they remain uncharged
 (B) they move farther apart
 (C) they move closer together
 (D) they remain negatively charged but unmoved

23. A positive point charge exerts a force of magnitude F on a negative point charge placed a distance x away. If the distance between the two point charges is halved, what is the magnitude of the new force that the positive point charge exerts on the negative point charge?

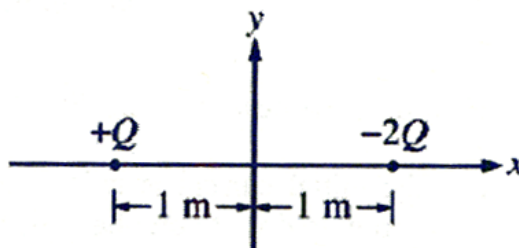
(A) $4F$ (B) $2F$ (C) $F/2$ (D) $F/4$

24. Two uniformly charged non-conducting spheres on insulating bases are placed on an air table. Sphere A has a charge $+3Q$ coulombs and sphere B has a charge $+Q$ coulombs. Which of the following correctly illustrates the magnitude and direction of the electrostatic force between the spheres when they are released?

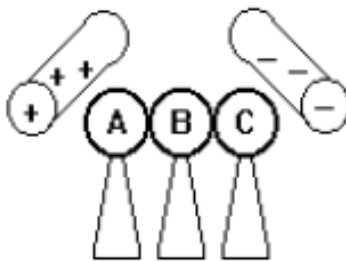


25. A person rubs a neutral comb through their hair and the comb becomes negatively charged. Which of the following is the best explanation for this phenomenon?

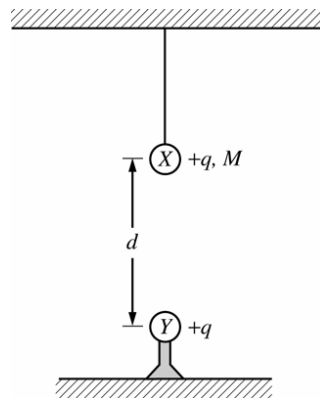
(A) The hair gains protons from the comb.
(B) The hair gains protons from the comb while giving electrons to the comb.
(C) The hair loses electrons to the comb.
(D) The comb loses protons to the person's hand while also gaining electrons from the hair.



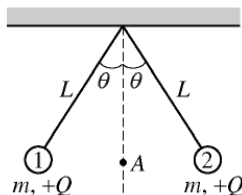
26. A charge of $+Q$ is located on the x -axis at $x = -1$ meter and a charge of $-2Q$ is held at $x = +1$ meter, as shown in the diagram above. At what position on the x -axis will a test charge of $+q$ experience a zero net electrostatic force?
- (A) $-(3 + \sqrt{8})$ m (B) $-1/3$ m (C) $1/3$ m (D) $(3 + \sqrt{8})$ m
27. Two point objects each carrying charge $10Q$ are separated by a distance d . The force between them is F . If half the charge on one object is transferred to the other object while at the same time the distance between them is doubled, what is the new force between the two objects?
- (A) $0.19 F$ (B) $0.25 F$ (C) $4.0 F$ (D) no change in F
28. Two identical spheres carry identical electric charges. If the spheres are set a distance d apart they repel one another with a force F . A third sphere, identical to the other two but initially uncharged is then touched to one sphere and then to the other before being removed. What would be the resulting force between the original two spheres?
- (A) $3/4 F$ (B) $5/8 F$ (C) $1/2 F$ (D) $3/8 F$



29. Three metal spheres A, B, and C are mounted on insulating stands. The spheres are touching one another, as shown in the diagram below. A strong positively charged object is brought near sphere A and a strong negative charge is brought near sphere C. While the charged objects remain near spheres A and C, sphere B is removed by means of its insulating stand. After the charged objects are removed, sphere B is first touched to sphere A and then to sphere C. The resulting charge on B would be of what relative amount and sign?
- (A) the same sign but $1/2$ the magnitude as originally on sphere A
 (B) the opposite sign but $1/2$ the magnitude as originally on sphere A
 (C) the opposite sign but $1/4$ the magnitude as originally on sphere A
 (D) the same sign but $1/2$ the magnitude as originally on sphere C



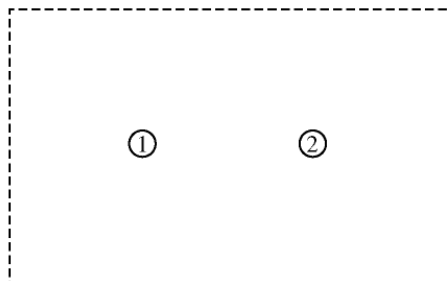
30. Sphere X of mass M and charge $+q$ hangs from a string as shown above. Sphere Y has an equal charge $+q$ and is fixed in place a distance d directly below sphere X . If sphere X is in equilibrium, the tension in the string is most nearly
- (A) Mg (B) $Mg - kq/d$ (C) $Mg + kq^2/d^2$ (D) $Mg - kq^2/d^2$



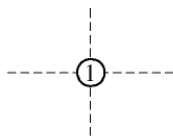
2. (10 points)

Two small objects, labeled 1 and 2 in the diagram above, are suspended in equilibrium from strings of length L . Each object has mass m and charge $+Q$. Assume that the strings have negligible mass and are insulating and electrically neutral. Express all algebraic answers in terms of m , L , Q , θ , and fundamental constants.

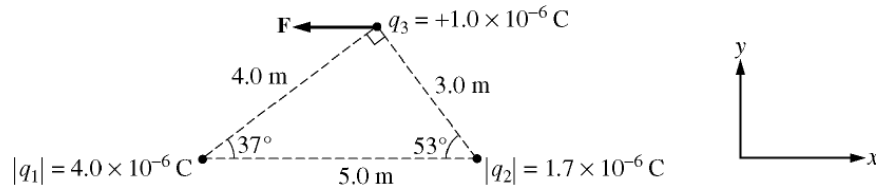
- (a) On the following diagram, sketch lines to illustrate a 2-dimensional view of the net electric field due to the two objects in the region enclosed by the dashed lines.



- (b) Derive an expression for the electric potential at point A , shown in the diagram at the top of the page, which is midway between the charged objects.
- (c) On the following diagram of object 1, draw and label vectors to represent the forces on the object.



- (d) Using the conditions of equilibrium, write—but do not solve—two equations that could, together, be solved for θ and the tension T in the left-hand string.



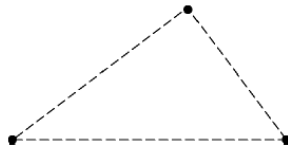
3. (10 points)

Three particles are fixed in place in a horizontal plane, as shown in the figure above. Particle 3 at the top of the triangle has charge q_3 of $+1.0 \times 10^{-6} \text{ C}$, and the electrostatic force \mathbf{F} on it due to the charge on the two other particles is measured to be entirely in the negative x -direction. The magnitude of the charge q_1 on particle 1 is known to be $4.0 \times 10^{-6} \text{ C}$, and the magnitude of the charge q_2 on particle 2 is known to be $1.7 \times 10^{-6} \text{ C}$, but their signs are not known.

(a) Determine the signs of the charges q_1 and q_2 and indicate the correct signs below.

q_1 ☐ Negative q_2 ☐ Negative
 ☐ Positive ☐ Positive

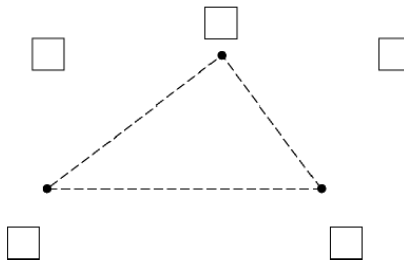
(b) On the diagram below, draw and label arrows to indicate the direction of the force F_1 exerted by particle 1 on particle 3 and the force F_2 exerted by particle 2 on particle 3.



(c) Calculate the magnitude of \mathbf{F} , the electrostatic force on particle 3.

(d) Calculate the magnitude of the electric field at the position of particle 3 due to the other two particles.

(e) On the figure below, draw a small \times in the box that is at a position where another positively charged particle could be fixed in place so that the electrostatic force on particle 3 is zero.



Justify your answer.

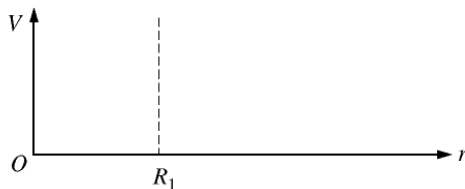
2. (15 points)

An isolated, solid copper sphere of radius $R_1 = 0.12$ m has a positive charge of 6.4×10^{-9} C.

(a)

- i. Calculate the electric potential at a point 0.10 m from the center of the sphere.
- ii. Calculate the electric potential at a point 0.24 m from the center of the sphere.

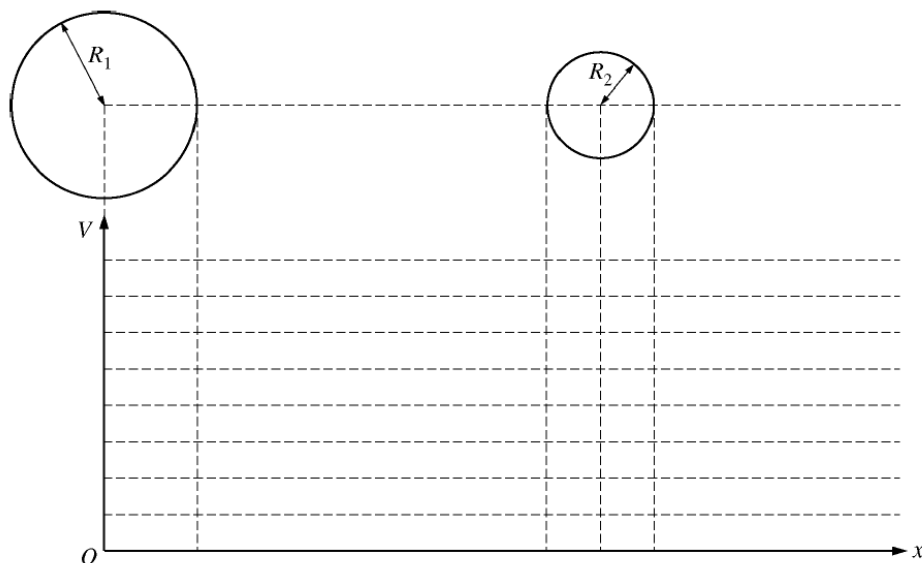
(b) On the axes below, sketch a graph of electric potential V versus radius r from the center of the sphere. Label the value at $r = 0$ on the vertical axis.

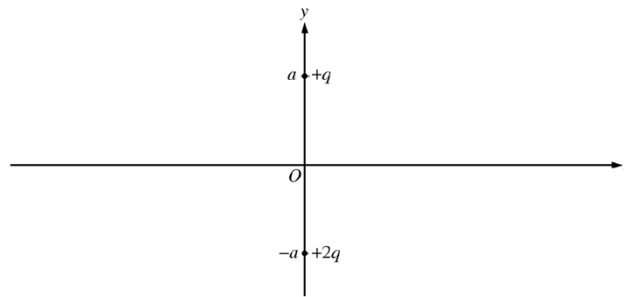


(c)

- i. Determine the magnitude of the electric field at a point 0.10 m from the center of the sphere.
- ii. Determine the magnitude of the electric field at a point 0.24 m from the center of the sphere.

(d) A second copper sphere of radius R_2 that is uncharged is placed near the first sphere, as represented in the figure below. On the axes below, sketch a graph of electric potential V versus distance along the x -axis shown, where the center of the first sphere is at $x = 0$.



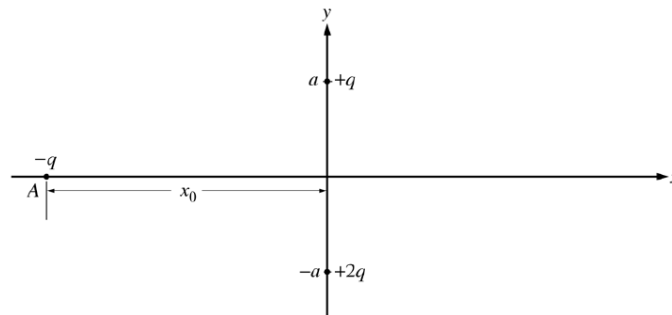


3. (15 points)

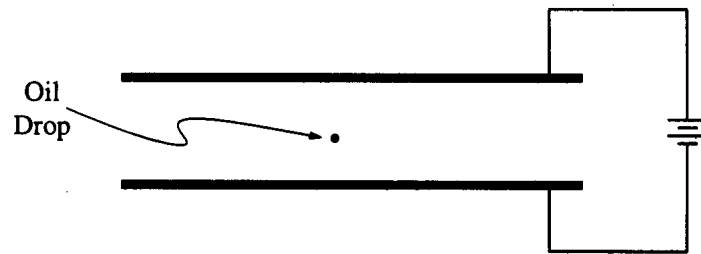
Two point charges are fixed on the y -axis at the locations shown in the figure above. A charge of $+q$ is located at $y = +a$ and a charge of $+2q$ is located at $y = -a$. Express your answers to parts (a) and (b) in terms of q , a and fundamental constants.

- Determine the magnitude and direction of the electric field at the origin.
- Determine the electric potential at the origin.

A third charge of $-q$ is first placed at an arbitrary point A ($x = -x_0$) on the x -axis as shown in the figure below.



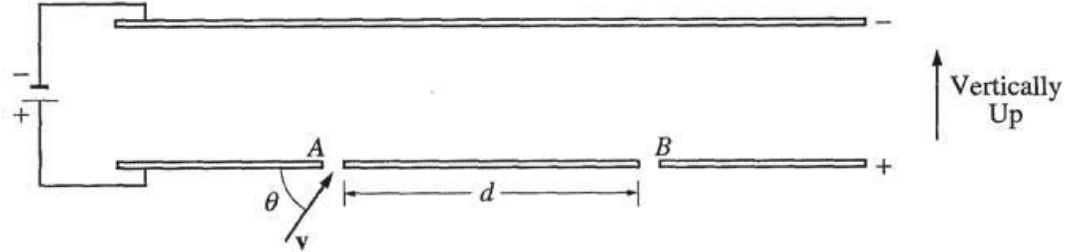
- Write expressions in terms of q , a , x_0 and fundamental constants for the magnitudes of the forces on the $-q$ charge at point A caused by each of the following,
 - The $+q$ charge
 - The $+2q$ charge



6. (15points)

Robert Millikan received a Nobel Prize for determining the charge on the electron. To do this, he set up a potential difference between two horizontal parallel metal plates. He then sprayed drops of oil between the plates and adjusted the potential difference until drops of a certain size remained suspended at rest between the plates, as shown above. Suppose that when the potential difference between the plates is adjusted until the electric field is $10,000 \text{ N/C}$ downward, a certain drop with a mass of $3.27 \times 10^{-16} \text{ kg}$ remains suspended.

- a. What is the magnitude of the charge on this drop?
- b. The electric field is downward, but the electric force on the drop is upward. Explain why.
- c. If the distance between the plates is 0.01 m , what is the potential difference between the plates?
- d. The oil in the drop slowly evaporates while the drop is being observed, but the charge on the drop remains the same. Indicate whether the drop remains at rest, moves upward, or moves downward. Explain briefly.



5. (15 points)

An electric field E exists in the region between the two electrically charged parallel plates shown above. A beam of electrons of mass m , charge q , and velocity v enters the region through a small hole at position A. The electrons exit the region between the plates through a small hole at position B. Express your answers to the following questions in terms of the quantities m , q , E , θ , and v . Ignore the effects of gravity.

- a.
 - i. On the diagram of the parallel plates above, draw and label a vector to show the direction of the electric field E between the plates.
 - ii. On the following diagram, show the direction of the force(s) acting on an electron after it enters the region between the plates.



- iii. On the diagram of the parallel plates above, show the trajectory of an electron that will exit through the small hole at position B.
- b. Determine the magnitude of the acceleration of an electron after it has entered the region between the parallel plates.
- c. Determine the total time that it takes the electrons to go from position A to position B.

- d. Determine the distance d between positions A and B.
- e. Now assume that the effects of gravity cannot be ignored in this problem. How would the distance where the electron exits the region between the plates change for an electron entering the region at A? Explain your reasoning.