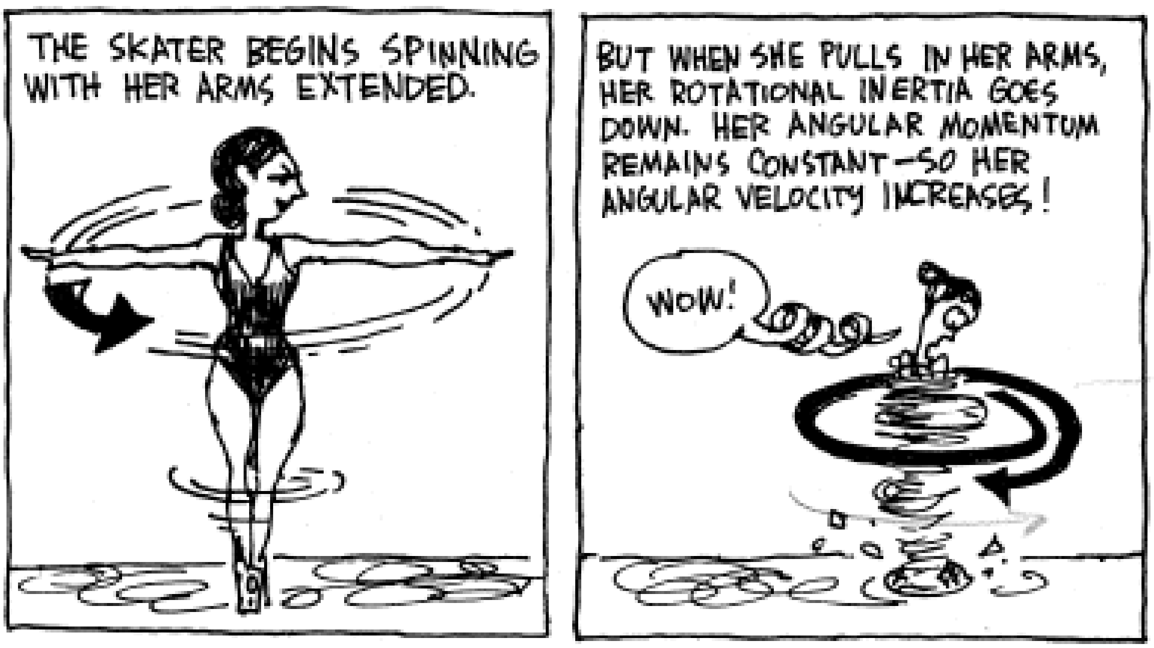
SALTUS GRAMMAR SCHOOL

AP Physics I



Unit 7 – Rotational Mechanics

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When things rotate a lot of physics happens. Rotating objects tend to keep rotating until their kinetic energy has been dissipated in much the same way that objects that move in a straight line (linear motion) do – they have rotational inertia, a resistance to change similar to linear inertia. Another property that is useful, as it is conserved, is the rotational equivalent to linear momentum, which is known as angular momentum.

7.1 Radians
7.2 Angular Velocity
7.3 Angular Acceleration
7.4 Rotational Kinematics
7.5 Torque and Rotational Dynamics
7.6 Rotational Energy
7.7 Angular Momentum

Sources for material:

Physics for Dummies, The Cartoon Guide to Physics, College Physics, 5 Steps to a 5

7.1 Radians

Objective:

- Understand what a radian is and why they are useful.
- Be able to convert between degrees and radians.

Question: What do the words RADIATION, RADIATE, RADIAN and RADIAL all have in common? Why?

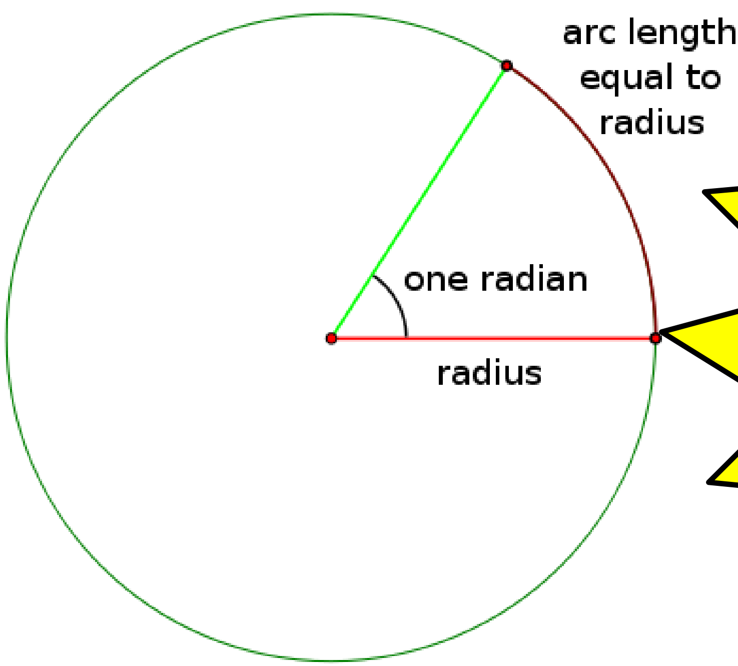
We all have grown up with the basic knowledge that there are 360 degrees in a circle. This was decided by the ancient Babylonians thousands of years ago and the exact reason has been lost to history. The two leading theories are:

- Roughly equal to the number of days in a year
- Has the largest number of divisors.

The degree has not always been universally used however. Sailors and navigators used 'points of a compass' for centuries as they could rarely measure or steer more accurately than a 1/32nd of a circle. The military used a finer measure called 'mils' and 'grads' were another attempt to metrify angles. The degree has become the de-facto standard and is likely to remain so for the foreseeable future. However, the degree is still arbitrary and mathematically awkward once we enter the realms of calculus and rotating bodies.

The 'natural' angular measure is the radian. They are not, however, intuitive or even particularly useful in the 'real' world, but they make the mathematics of angles and rotation considerably easier – so it is worth learning them and getting used to using them.

A radian (rad) is the angle that occurs when the arc of a circle subtended by the angle is exactly equal to the radius. Therefore, as the circumference is $2\pi r$, there must be 2π radians in a circle of 360°. This seems to be odd as of course π is an irrational number! NOTE: the unit is **"rad"** and NEVER **"rads"** – as the "s" can be confused for seconds.



TIP: remember to check that your calculator is appropriately set to either RAD or DEG when using angles.

To convert between degrees and radians it is helpful (and easiest) to remember that the ratios of the angles to that of a full circle must always be the same:

$$\frac{\theta_{rad}}{2\pi} = \frac{\theta_{deg}}{360°}$$

For example: an angle of 36°in radians is:

$$\frac{\theta_{rad}}{2\pi} = \frac{36^{\circ}}{360°} \rightarrow 2\pi\frac{36°}{360°} = 0.63\ rad$$

Example 1

What is 23° in radians?

Example 2

What is 90° in radians, expressed as either a fraction or a decimal?

Example 3

How many degrees are 1 radian?

7.2 Angular Velocity

Objectives:

- To understand the defining variable for constant rotation; period, frequency, tangential velocity and angular velocity.
- To be able to convert from one variable to another in a variety of ways.
- To be able to measure the angular and tangential velocity of a rotating object.

Just like linear motion can be described by a set of kinematic variables, the motion of a rotating object can be described by similar variables. Some of these have been met before in Unit 3 - Torque and Circular Motion.

Period

The period, *T*, is the time taken to complete one rotation and is measured in seconds.

Frequency

The frequency, *f*, is the reciprocal of the period. It is the number of rotations per second. The units are usually expressed as Hertz, Hz, or in SI units as either 1/s or s^{-1}. The relationship between the frequency and the period is, by definition,

$$f = \frac{1}{T}$$

A more common unit in the 'real' world is the number of revolutions per minute (rpm).

Tangential speed or velocity

The tangential speed or velocity is the speed or velocity vector of the rotating object at any given point in time around the circle. Assuming that the speed is constant, it can be found by remembering the simple relationship between speed, distance and time:

$$velocity = \frac{distance}{time} = \frac{circumference}{period}$$

$$v = \frac{2\pi r}{T}$$

Example 4

Calculate the tangential velocity of the Earth in its orbit around the Sun. (Earth's orbital radius is approximately 150 million km)

Angular Velocity

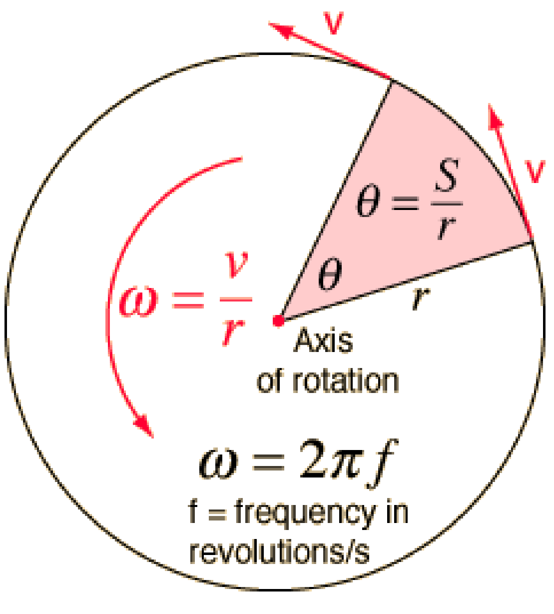
A very useful variable is the angular velocity denoted by the lower case Greek letter omega, ω. It is the rotational equivalent of linear velocity and is defined as the rate of change of angle:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

The units are either degrees/second or radians/sec (rad/s or rads^{-1})

** So, now you know why you need to be careful with the unit "rad" that as referred to earlier!**

A very common real-life measure of angular velocity or speed is the revolutions per minute – or rpm. It is useful to be able to convert this!



Example 5

The old vinyl records had rates of revolution of either 45 rpm for singles or 78 rpm for the really old albums. Calculate the angular velocity of these in terms of degrees/second and rad/s.

Example 6

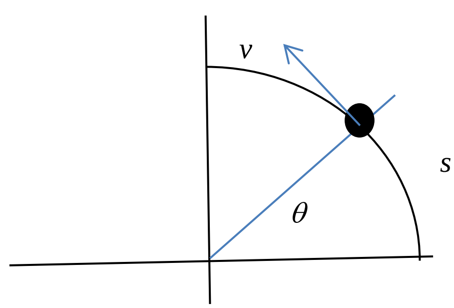
The Moon takes about 27.3 days to orbit the Earth. What is its angular velocity?

Example 7

A satellite is orbiting at 8.7×10^{-4} rad/s. How long will it take to orbit the world?

Converting between Angular Velocity and Tangential (Linear) Velocity

It is important to note that the following equations only work if the angle is measured in RADIANS!



In radians, $s = r\theta$, so the velocity that it travelled along the arc of distance, s, in time, t, is:

$$v = \frac{s}{t} = \frac{r\theta}{t}$$

Given that angular velocity, $\omega = \frac{\Delta\theta}{\Delta t}$, we get:

$$v = r\omega$$

Question: What happens to the linear velocity of an object rotating at a constant rate as the radius tends towards zero? (in other words, if an ant was slowing walking towards the centre of a record that was spinning, was would happen to its tangential velocity?)

Example 8

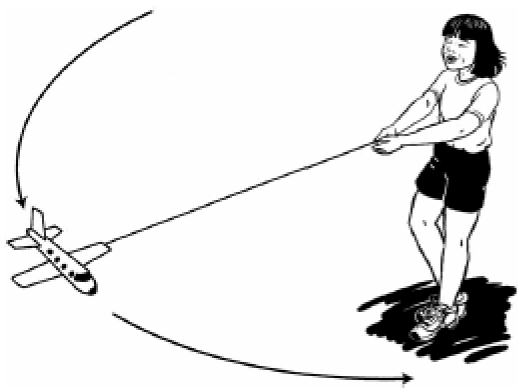
A ball on a string is going around in a circle at 6.0 rad/s. What is its tangential velocity if the radius of the circle is 2.0 m? (ans = 12 m/s)

Example 9

A satellite is orbiting the Earth, which has an average radius of 6370 km, at an altitude of 200 km and an angular speed of 1.17×10^{-3} rads^{-1}. What is the satellite's tangential speed in km/hr? (ans = 27,673 km/hr)

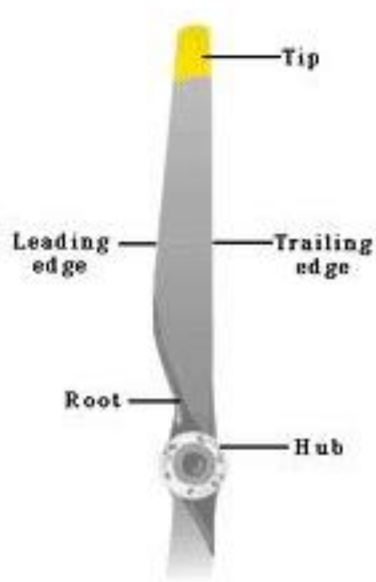
Example 10

You are flying a toy plane on a string, and it is going around you at 8 m/s at a distance of 10 m. What is its angular speed in radians/second? (ans = 0.8 rad/s)



Example 11

The tip of an airplane propeller is going at 500 km/hr. If the propeller has a radius of 1.5 m, what is its angular speed?

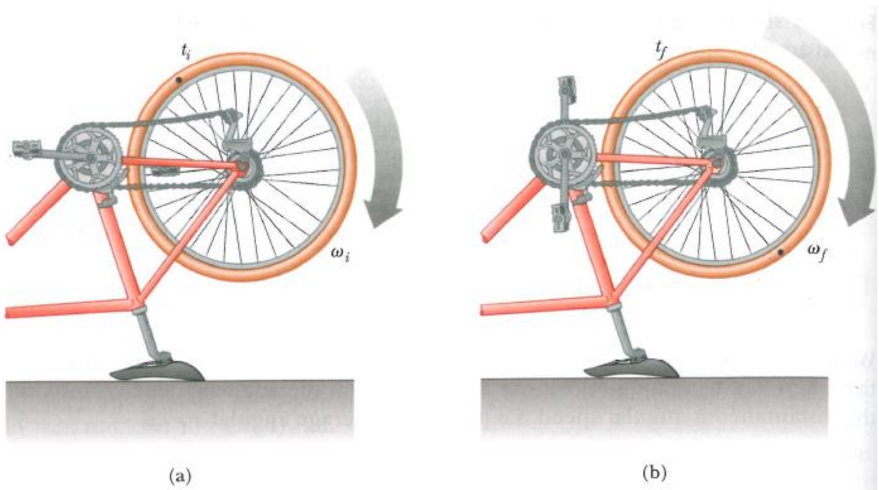


Experimental Considerations: Describe, with diagrams, some of the methods that we could use to measure the rotational speed, of a wheel, in the physics lab. You have access to all the usual equipment.

7.3 Angular Acceleration

Objectives:

- To understand the concept of angular acceleration as the rate of change of angular velocity
- To be able to calculate and measure the angular acceleration of a rotating object
- To be able to interchange between angular acceleration and tangential acceleration.



Just as with linear motion, you can have angular acceleration, *a,* when you are dealing with rotational motion. For example, if the spin of an object increases or decreases, as shown in the diagram above where the angular velocity increases from ω_i to ω_f in ($t_f - t_i$) seconds. In linear acceleration, the equation for acceleration is:

$$a = \frac{\Delta v}{\Delta t}$$

As with all the other equations of motion, we only need to substitute the correct equivalent angular variable for the linear one:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

The unit of angular acceleration is radian/second^2 or rad/s^2.

Example 12

Your toy plane on a string that is buzzing around your head accelerates from 2.1 rad/s to 3.1 rad/s in 1.0 s. What is its angular acceleration? (ans = 1 rad/s^2)

Example 13

A model globe is turning at 2.0 rad/s, which is not fast enough. It is given a push, which accelerates it to 5.0 rad/s in 0.1 s. What is its angular acceleration? (ans = 30 rad/s^2)

Question: How does the angular acceleration of a bicycle when relate to the rim of the wheel's linear acceleration? How does this relate to the acceleration of the bicycle in general?

Question: How could we measure the angular acceleration of a wheel in the lab?

7.4 Rotational Kinematics

Objectives:

- To be able to compare the linear and rotational kinematic equations of motion
- To be able to perform calculations using the equations and solve problems

The basic idea is that the linear kinematics equations can be replicated for rotational motion by substituting key variables. This makes life much simpler!

Linear equation: $x = v_o t + {}^1/_2\, at^2$

Rotational equation: $\theta = \omega_o t + {}^1/_2\, \alpha t^2$

Linear equation: $v^2 = v_o^2 + 2ax$

Rotational equation: $\omega^2 = \omega_o^2 + 2\alpha\theta$

Variable	**Linear**	**Rotational equivalent**
Displacement		
Velocity		
Acceleration		

Example 14

A marble is rolling around a circular track at 6.0 rad/s and then accelerates at 1.0 rad/s^2. How many radians has it gone through in 1 minute? (ans = 2160 rad)

Example 15

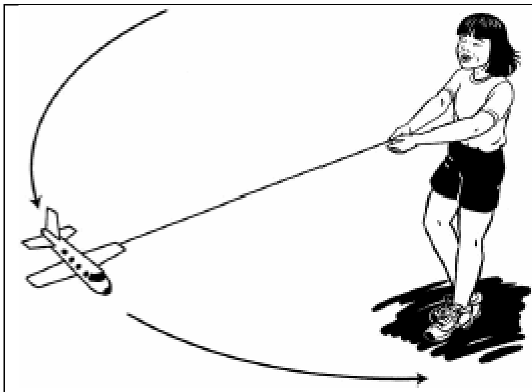
You are whipping a ball on a string around in a circle. If it is going 7.0 rad/s and after 6.0 s has gone through a total angular distance of 60.0 rad as it accelerated, what was its angular acceleration? How many revolutions did it take to accelerate? (ans = 1.0 rad/s^2)

Example 16

A merry-go-round slows down from 6.5 rad/s to 2.5 rad/s, undergoing an angular deceleration of 1.0 rad/s^2. How many radians does the merry-go-round go through during this slow down? How many revolutions is this? (ans = 18 rad)

Example 17

You are still flying the toy plane! It is going around at 20 m/s, 10 m from you. If it accelerates to a final velocity of 30 m/s in 80 s, what is its angular acceleration? (0.013 rad/s^2)

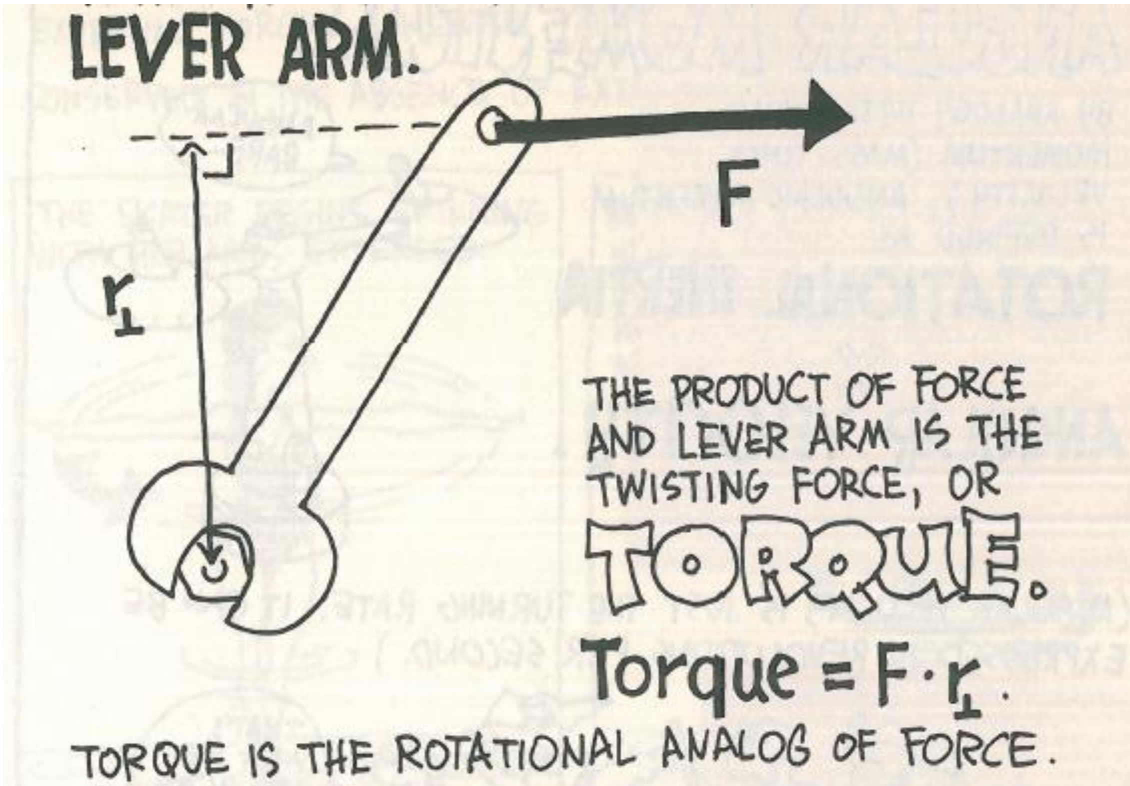


Question: Roll a wheel along the ground. How do the rotational variables relate to the linear motion of the wheel?

7.5 Torque and Rotational Dynamics

Objectives:

- Understand the concept of torque
- Know the effect of applying a torque to an object
- Understand the concept of rotational inertia
- Be able to use Newton's Second Law for rotating objects



Just as with linear motion, dynamics is the study of how force(s) changes the motion. When considering the force needed to accelerate or decelerate a rotating object we need to consider the torque or turning moment applied by the force rather than the force itself. This was discussed in Unit 3 – Torque (Moments). The torque due to a force, F, at a radian (perpendicular) distance, r, from a pivot (or centre of rotation) is defined as:

$$\tau = Fr$$

The force needed to accelerate a ball moving in a circle is given by $F = ma$, so the torque:

$$\tau = Fr = mra$$

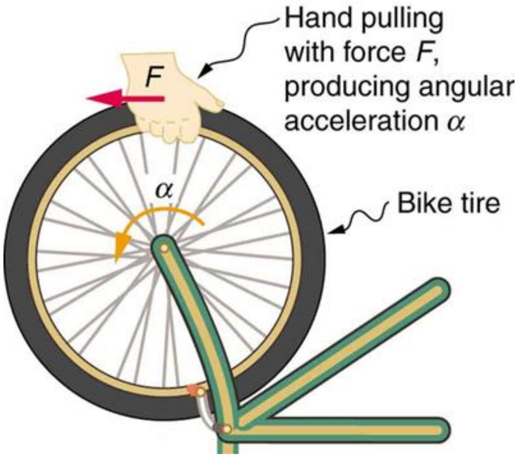
And since the acceleration is $a = ra$, we get:

$$\tau = mr^2\alpha$$

This is important as it relates torque to the angular acceleration. The quantity mr^2 is called the moment of inertia *and it represents the effort required to change the rotation of an object. It is given the symbol I.* So the torque becomes:

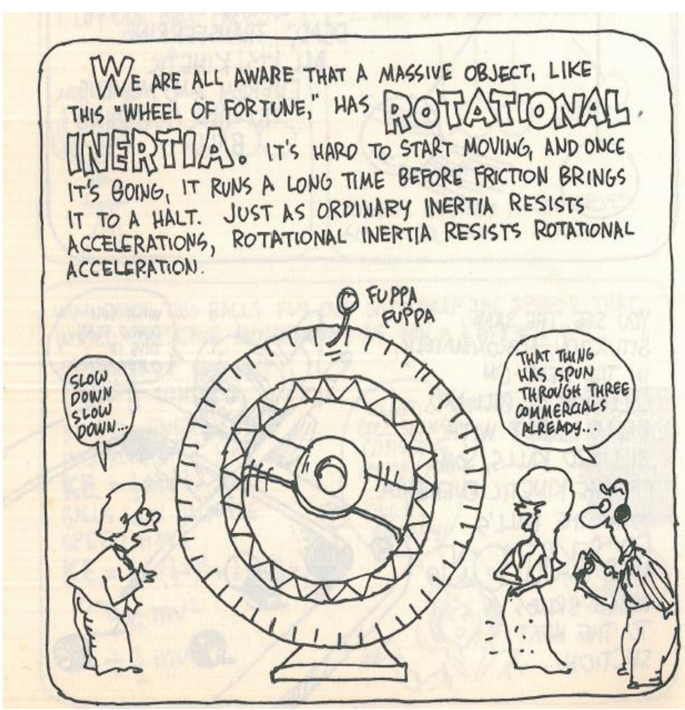
$$\tau = I\alpha$$

This looks strange as most of the letters are Greek! But it is really the rotational version of Newton's Second Law. Recall that mass is a measure of resistance to change of motion, i.e. inertia….



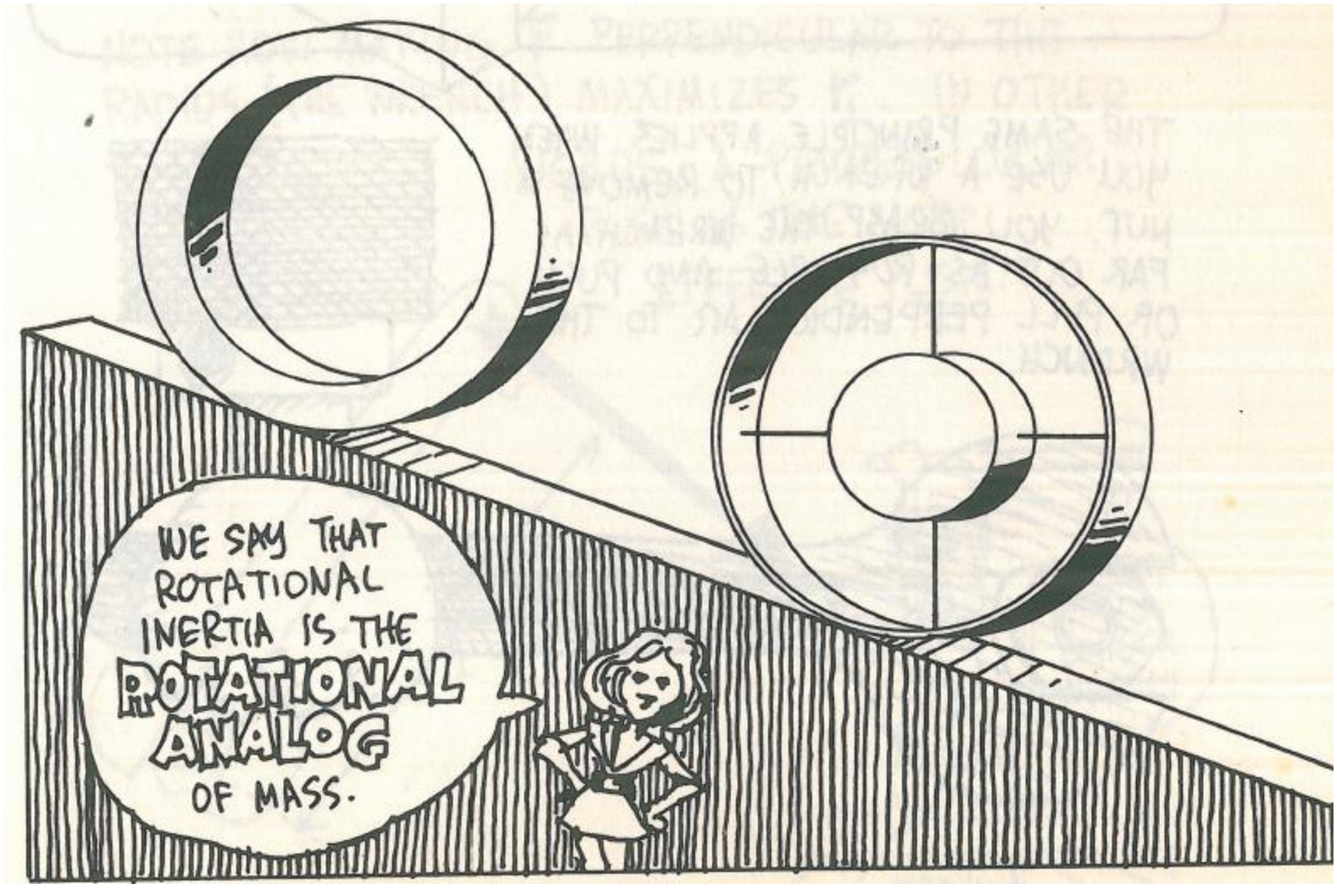
Linear Quantity	Rotational Equivalent
Force	
Mass (inertia)	
Acceleration	

Question: What is the moment of inertia in physical terms?



The moment of inertia of a ball on a thin string is given by $I = mr^2$. However, things are not as simple as all that! The moment of inertia varies dramatically as the geometry of the object changes. It is different for a bike wheel where the bulk of the mass is located far from the central pivot, so the torque required is high, compared to that of a solid cylinder where the mass is more concentrated to the centre, so therefore less of a torque is required as the effective radius is reduced.

Rotational Inertia is just another way of saying Moment of Inertia



Description	Diagram	Moment of Inertia
Disk		
Hollow cylinder		

Hollow sphere		
Hoop		
Solid cylinder		
Solid sphere		

The closer the radial centre of mass is to the centre – the lower the rotational inertia is.

Example 18

A solid cylinder of mass 5.0 kg is rolling down a ramp. If it has a radius of 0.1 m and an angular acceleration of 3.0 rad/s^2, what torque is operating on it? (ans = 0.075 Nm)

Example 19

A tyre with a radius of 0.50 m and mass of 1.0 kg is rolling down the street. If it accelerating with an angular acceleration of 10.0 rad/s^2, what torque is operating on it? (ans = 2.5 Nm)

Example 20

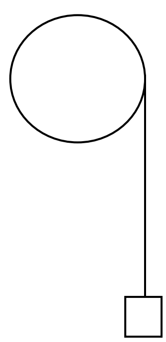
You are spinning a 2.0 kg hollow ball with a radius of 0.50 m, starting from rest and applying a 12.0 N torque. What are the angular and tangential speeds after 10.0 s? (ans = 360 rad/s)

Example 21

You are throwing a 0.3 kg disc with a radius of 0.1 m, accelerating it with an angular acceleration of 20.0 rad/s^2. What torque was applied? (ans = 0.003 Nm)

Example 22

A 0.4 kg mass is hanging from a wheel of radius 0.15 m as shown. The mass falls 2.0m from rest in 6.5 s. a) draw in the forces and label the key variables, b) why does the mass not accelerate at 9.81 m/s2?, c) calculate the vertical acceleration of the falling mass, d) calculate the rotational inertia of the wheel.

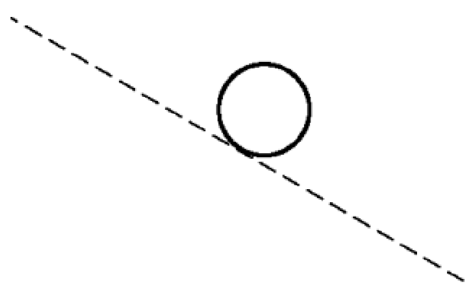


7.6 Rotational Kinetic Energy

Objectives:

- Understand that a rotating body is moving and therefore has kinetic energy
- Be able to calculate the kinetic energy of rotation
- Be able to solve problems involving the conservation of energy (such as an object rolling down a slope)

Question: What are the forces that are acting on a ball that is on a slope? Which force or forces cause the ball to roll? Explain.



When you apply a force to move an object in linear mechanics you do work, as we discussed in Unit 5 – Work, Energy and Power.

$$Work = force \times distance$$

The same is true for rotational motion, in order to change the angular velocity of a rotating body you need to put in some energy and do work. As with the kinematics and dynamics the simplistic way of looking at it is to substitute the rotational variables for the linear ones in the relevant equations.

When a force is applied to the edge of a tyre to get a wheel moving, the distance equals the radius multiplied by the angle that the tyre turns, $s = r\theta$, so:

$$W = Fs = Fr\theta$$

But as the torque, τ, is Fr, you are left with:

$$W = Fs = Fr\theta = \tau\theta$$

Which is the rotational analogue of the linear equation $W = Fx$!

Example 23

How much work do you do if you apply a torque of 6.0 Nm over an angle of 200 radians? (ans = 1200J)

Rotational Kinetic Energy

The kinetic energy of an object moving in a linear motion is defined as the work required to bring it to a stop. The equation is the (hopefully!) well-known:

$$KE = {}^1/_2\, mv^2$$

Using the idea that rotational variables can be substituted for the linear ones, we get the kinetic energy of a rotating object to be:

Example 24

A 100 kg solid sphere of radius 1.0 m is spinning with an angular velocity of 10.0 rad/s. What is its rotational kinetic energy? (ans = 2000J)

Example 25

How much work do you need to do to spin a hollow sphere of mass 10 kg and a radius of 0.5 m from rest to 100 rpm? (ans = 91.9 J)

Example 26

How much rotational kinetic energy does a 12 kg tyre of radius 0.8 m that is spinning at 100 rpm have? (ans = 211.7 J)

We can further extend this idea to include the conversion of PE to KE as an object rolls down a slope! In the past, for example in Unit 5 – Work, Energy and Power, we used the conservation of energy to calculate the speed of a falling object:

$$PE_{lost} = KE_{gained} = mgh = {}^{1}/_{2}\, mv^2$$

However, when an object ROLLS down a slope, it has rotational kinetic energy IN ADDITION to the normal linear kinetic energy. So by including the rotational KE we get:

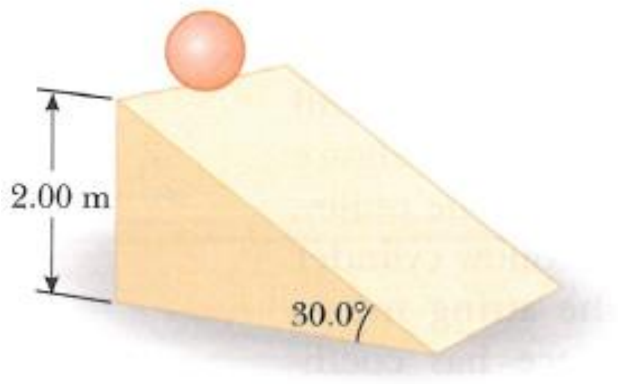
$$PE_{lost} = KE_{gained} + KE_{rotational}$$

Having the two types of velocity, linear and angular, in the same equation is a bit messy, so we can simplify it by using the relationship that we used earlier – assuming that the tangential speed is equal to the rotational speed *, i.e. that $v = r\omega$.

So….

Question: Under what conditions do you think that the situation above (marked by *) would be untrue?

Example 27



A ball of mass 2 kg and radius 0.2 m rolls down a 30° slope. What is the linear speed of the ball at the bottom of the slope?
(ans = 5.3 m/s)

Example 28

If a hollow cylinder is located at the top of a 4.0 m high ramp, what is its speed when it reaches the bottom? (ans = 6.3 m/s)

Example 29

A basketball is rolling down a ramp, starting at a height of 4.8 m. What is the speed when it reaches the bottom of the ramp? (ans = 7.5 m/s)

Question: A block of ice and a ball are released from the top of a ramp. Which hits the bottom first and why?

7.7 Angular Momentum

Objectives:

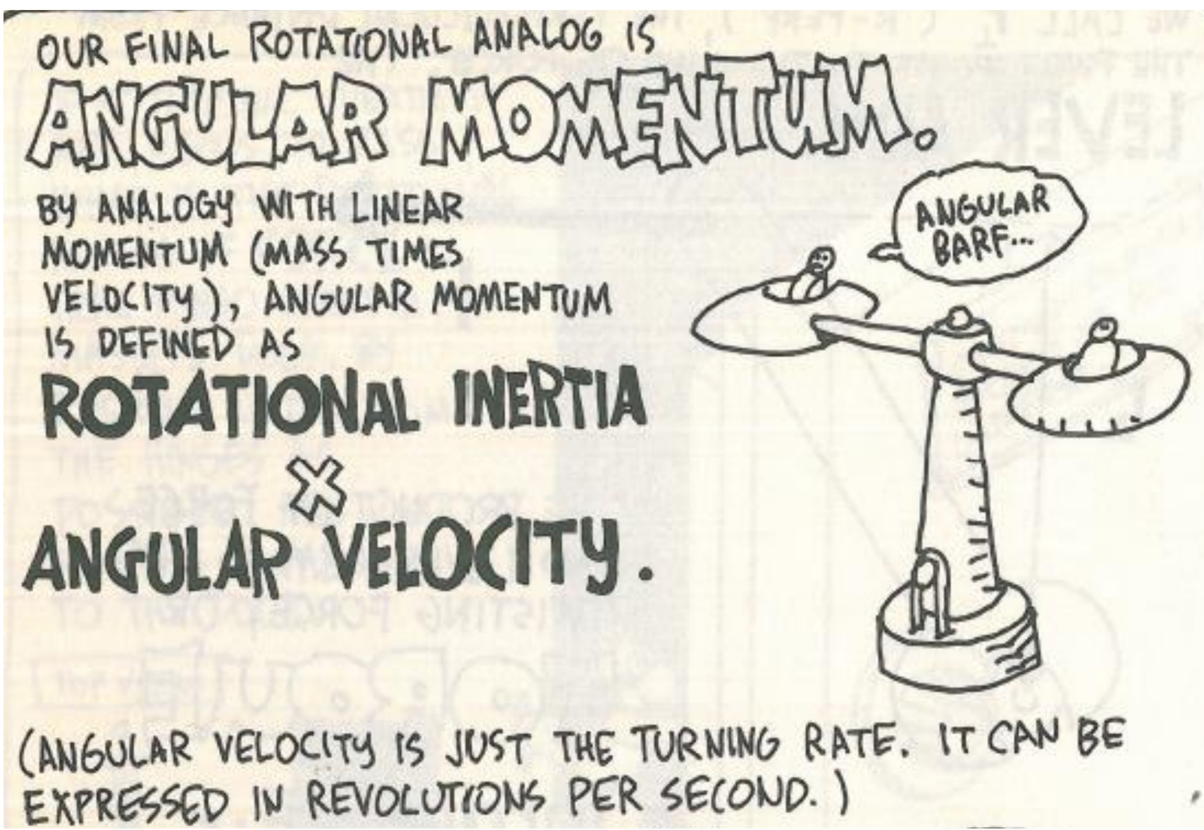
- Understand the concept of angular momentum
- Understand the implications of the conservation of angular momentum
- Be able to solve problems involving the conservation of angular momentum

In Unit 6 - Linear Momentum we learned about an important property of matter called linear momentum, p, which is conserved during collisions and explosions. The rotational equivalent, L, pretty much does the same job and can have some interesting consequences – especially in space!

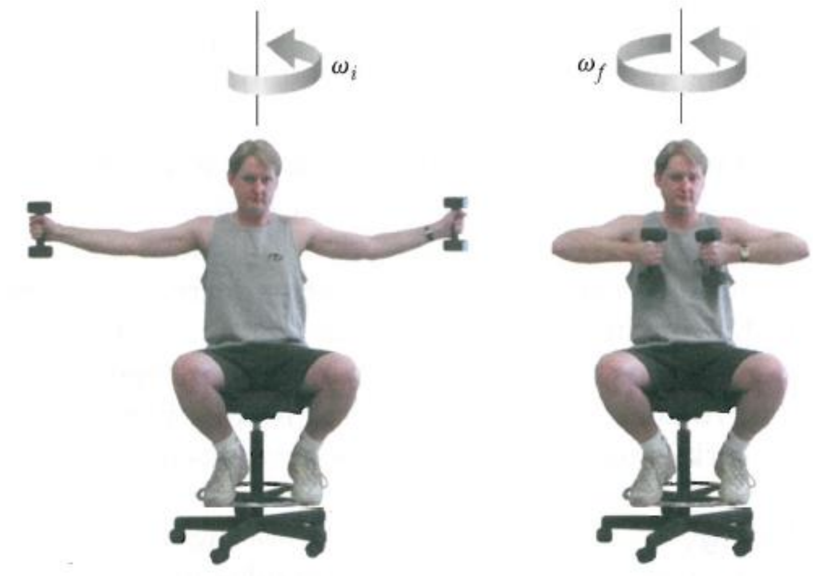
$$p = mv$$

So the equation for angular momentum is:

In physics, angular momentum is conserved – a rotating body wants to keep rotating with the same angular momentum. If the moment of inertia changes then the angular speed must change to compensate! This leads to the famous ice skater cartoon that can be found on the front cover. It also explains why planets keep spinning, as does the Sun.



a) The student is given an initial angular speed while holding two masses out.
b) The angular speed increases as the student draws the masses inwards.



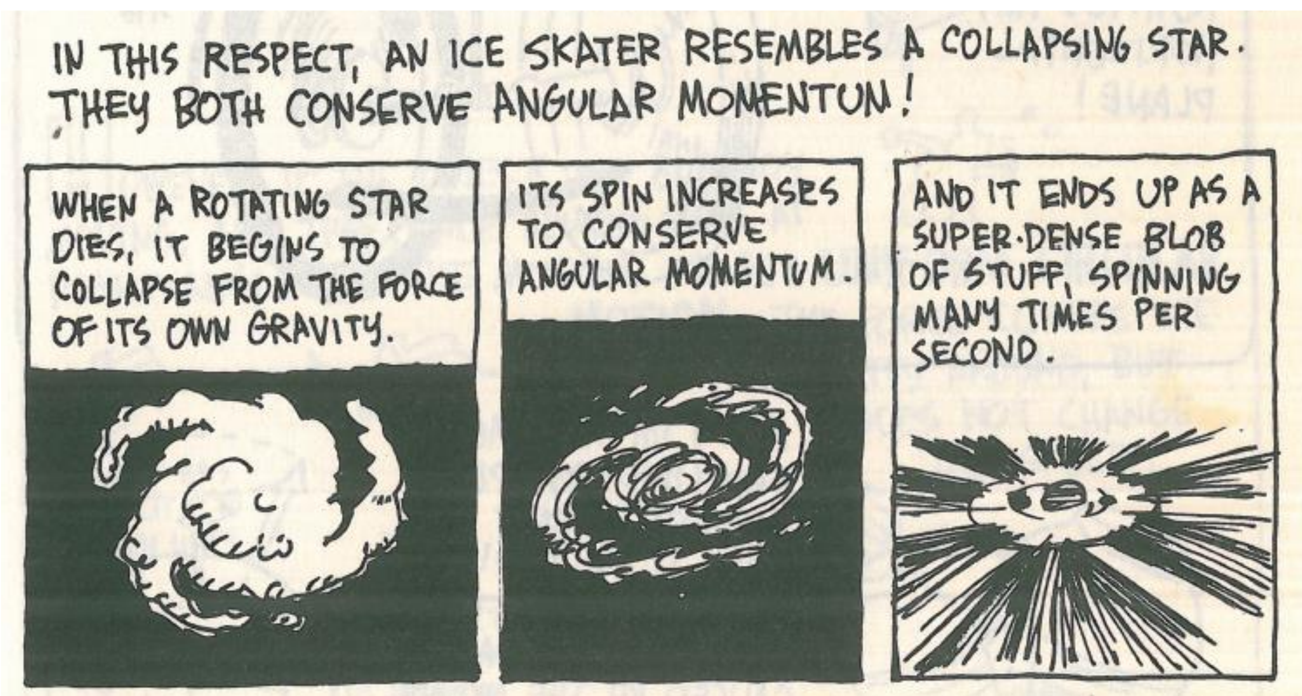
(a) (b)

Example 30

If a 500 kg merry-go-round with a radius of 2.0 m is spinning at 2.0 rad/s and a boy with a mass of 40 kg jumps on the outer rim, what is the new angular speed of the merry-go-round? (ans = 1.7 rad/s)

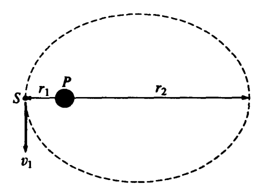
Example 31

A 2000 kg space station, which is a hollow cylinder with a radius of 2.0 m, is rotating at 1.0 rad/s when an astronaut with a mass of 80 kg lands on the outside of the station. What is the station's new angular speed? (ans = 0.96 rad/s)



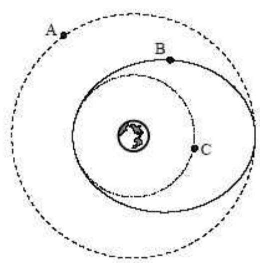
Question: Explain this cartoon!

Angular Momentum in Orbital Questions



1. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?

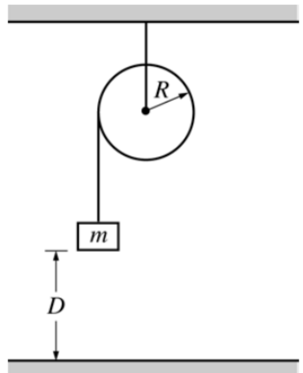
   (A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $\frac{r_1+r_2}{2}v_1$ (D) $\frac{r_2-r_1}{r_1+r_2}v_1$



2. Three equal mass satellites A, B, and C are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B, and L_C. Which of the following statements is correct?

   (A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

Some AP Style Questions



I have changed this from an algebra question to a calculation question. Add the given values to the diagram!

1. A solid disk of unknown mass and known radius R = 0.8 m is used as a pulley in a lab experiment, as shown above. A small block of mass m = 0.5 kg is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block is released from rest and takes a time t = 3.2 s to fall the distance D = 2.0 m to the floor.

a) Will the falling mass accelerate at a rate faster, slower or the same as that of free-fall? Explain. (2)

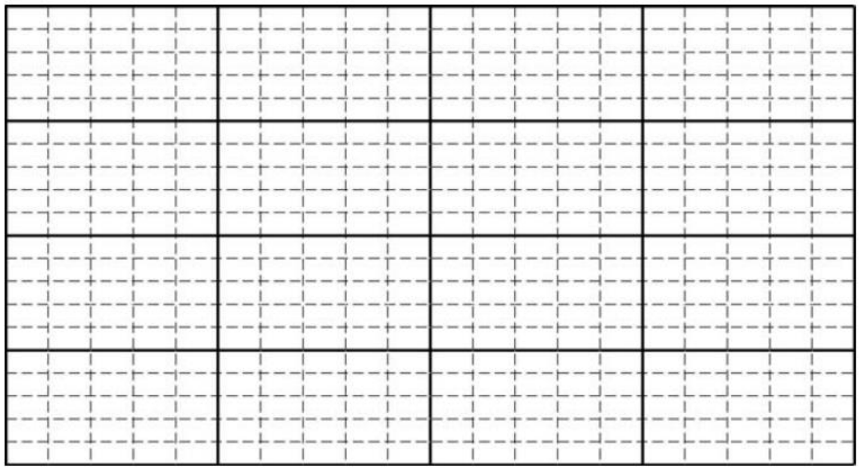
b) Calculate the linear acceleration a of the falling block in terms of the given quantities. (2)

c) In a more physicy investigation, the time t is measured for a range of various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i) What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning. (2)

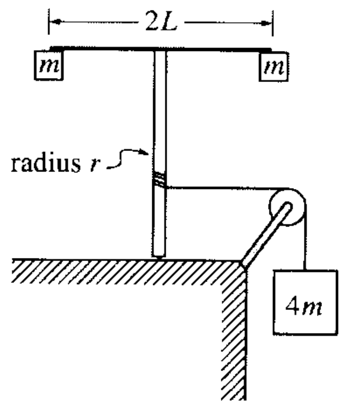
ii) On the grid below, plot the quantities determined in b) i), label the axes, and draw the best-fit line to the data. (3)



iii) Use your graph to calculate the magnitude of the acceleration. (2)

d) Calculate the rotational inertia of the pulley. (2)

e) The value of acceleration found in b) iii) along with numerical values for the given quantities and your answer to c), can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy. (2)



Experiment A

2. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius *r*, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a) Will the block of mass $4m$ descend at free-fall? Explain. (2)

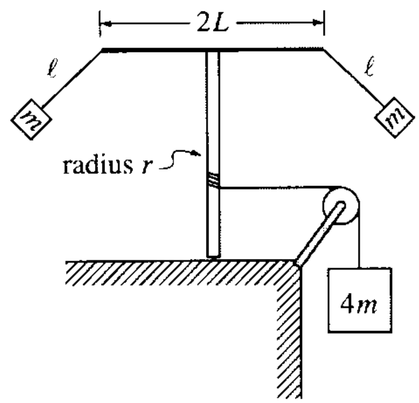
b) Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole. (2)

c) Determine the downward acceleration of the large block. (2)

d) Outline one method that could measure this acceleration in the physics lab. (2)

e) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare with the value $4mgD$? Check the appropriate space below and justify your answer. (2)

Greater than $4mgD$ _______ Equal to $4mgD$ _____ Less than $4mgD$ _______

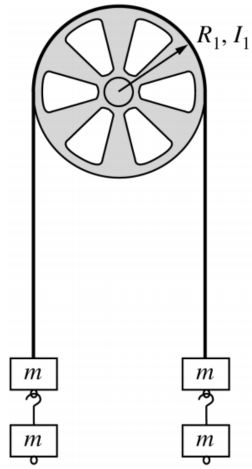


Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length l. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

f) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c)? Check the appropriate space below and justify your answer. (2)

Greater before ____ Equal to before ____ Less than before ____



I have modified this question to be an Atwood Machine question that compares the Unit 2 approach of ignoring the inertia of the pulley to the Unit 7 approach that includes it. I have again made it a numerical question.

3.

Part 1 – Ignoring the rotational inertia of the pulley (Unit 2)

An Atwood Machine is set up over a pulley of negligible friction and mass with 4 masses each of 0.2 kg. One of the masses is removed from the left side and added to the right side.

a) Draw two free body diagrams of the left and right sides. Be sure to clearly show the relative sizes of the forces. (2)

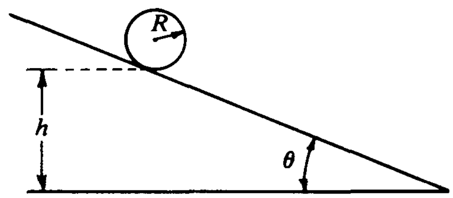
b) Determine the acceleration of the falling masses. (3)

Part 2 – taking the rotational inertia of the pulley into account (Unit 7)

A pulley of radius R_1 = 0.4 m, mass = 1.0 kg and rotational inertia I_1 = 0.08 kgm^2 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass 0.2 kg attached to either end, as shown above. Assume that the cord does not slip on the pulley. As before, one block is now removed from the left and hung on the right. When the system is released from rest, the three blocks on the right accelerate downward.

c) Does the system accelerate faster, the same or slower than before? Explain your reasoning. (2)

a) Determine the acceleration of the falling masses. (3) (hint: find the tension in the string as a function of the tangential acceleration of the pulley, then proceed using the simultaneous equations for Newton's Second Law on the blocks)



3. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height h above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express relevant answers in terms of M, R. h, g, and θ.

a) Determine the speed of the sphere when it is at the bottom of the plane (3)

b) The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

i) How does the speed of the hollow sphere at the bottom of the plane compare to that of the solid sphere? Explain your reasoning. (2)

ii) State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer. (2)